Controller Coverings and Switched Control Systems*

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Abstract

In its simplest form, the *controller covering problem* is as follows:

Given a subset $\mathcal{M}$ of admissible process model transfer functions in a metric space $\mathcal{T}$ equipped with a metric $\mu(\cdot,\cdot)$, find if possible a **finite** *set of controller transfer functions*

$\mathcal{K}$ with the property that, for each transfer function $t \in \mathcal{M}$, there is at least one controller transfer function $\kappa \in \mathcal{K}$ which endows the closed-loop system transfer function $\frac{t\kappa}{1+t\kappa}$ with at least stability and possibly other prescribed properties. Such families $\mathcal{K}$ are called **controller covers** *of $\mathcal{M}$.*

Conventional linear feedback theory typically starts with the assumption that the transfer function of the process to be controlled lies in a single ball $\mathbb{B}\{\nu, r\}$ in $\mathcal{T}$ of radius r centered at nominal transfer function ν, where r and ν are known and fixed. On the other hand, main stream parameter-adaptive control tries to deal with the more general general situation in which the process model family is of the form

$$\mathcal{M} \stackrel{\Delta}{=} \bigcup_{p\in\mathcal{P}} \mathbb{B}\{\nu_p, r_p\}$$

where $\mathcal{P}$ is a compact continuum {or more generally a finite union of compact continuums}

within a finite dimensional normed linear space, ν_p is the *pth nominal* transfer function, $p \longmapsto r_p$ is a bounded, positive function on $\mathcal{P}$ and $\mathcal{N} \stackrel{\Delta}{=} \{\nu_p : p \in \mathcal{P}\}$ is a compact family of *nominal* transfer functions. The first step in the development of an adaptive control for such a transfer function family is typically to define {if possible}, for each $p \in \mathcal{P}$, a controller with transfer function k_p,

capable of adequately controlling each transfer function in $\mathbb{B}\{\nu_p, r_p\}$. Adaptive control is then carried out by applying to the process a parameter-varying controller with 'instantaneous ' transfer function κ_σ where $\sigma(t)$ is {in accordance with certainty equivalence}

the best estimate at time t of the index of the ball within which the actual process model transfer function resides.

An alternative approach would be to try {if possible} to develop a finite controller cover of $\mathcal{M}$; the idea then would be to switch among these controller from time to time in accordance with certainty

equivalence. There are a number of good reasons for considering this alternative: Parameters may enter nominal process model transfer functions in complicated

ways making estimation over the continuum intractable. The step of actually constructing a *continuum* of well-designed controllers for a continuum of process models is at best challenging and certainly constraining, especially if the controllers are to be designed using linear quadratic or H_∞ techniques. In the multivariable case these issues of course would be even more severe - and in the nonlinear case,

the parameter-dependent nominal process models {in this case dynamical systems} would have to satisfy a certain 'separability condition' which, for example, nonlinear parameter-dependent models as simple as $\dot{y} = \sin(py) + u$, fail to satisfy .

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The aim of this talk is to discuss the controller covering problem, and how its solutions might be used to develop switched adaptive algorithms for controlling poorly modeled processes.