Existence of standing gravity waves in deep water

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This is a joint work with Pavel Plotnikov (Novosibirsk) and John Toland (Bath).

We consider the two-dimensional potential flow of *standing gravity waves on an infinitely deep water*, with *no surface tension at the free surface*. This problem is *completely resonant*: linear theory gives *infinitely many eigenmodes for any rational value of the dimensionless parameter* $\mu = gT^2/2\pi\lambda$ (T and λ being time and space periods). The existence of infinitely many *approximate solutions* at any order is known, under the form of power series of the amplitude, where there is an infinite choice for the leading modes (one or several). We use a formulation of Dyachenko *et al* [1] leading to a nonlocal second order partial differential equation. For proving the existence of solutions with a given asymptotic expansion, we use an appropriate version of the Nash-Moser implicit function theorem, where the major difficulty is to invert the linearized operator near a perturbation of an a priori chosen approximate solution (see [2]). After changing coordinates and an adapted averaging on a non local operator, the linearized operator becomes first order and non-local in space and second order in time. Moreover, its main part is diagonal with constant coefficients and the remainder is regularizing, or quasi-one-dimensional. Then, there are two difficulties for inverting this linear operator: i) its restriction to the infinite-dimensional kernel of the linearization at zero, may be well estimated and its inverse exists provided that a slight condition $H(I)$, on the finite set I of integers designing the leading modes, is satisfied (see [3]). ii) There is a small-divisor problem arising on the complement of this kernel, which can be overcome by considering only particular values of the amplitude, according to their Diophantine properties. For most choices of subsets I of integers for the finite number of dominant modes, this yields to the *existence of a set of multimodal standing waves* corresponding to a set of values of $\mu > 1$ *w*hich is dense at 1 (1 is a Lebesgue point of this set).

References

[1] A.I.Dyachenko, E.A.Kuznetsov, M.D.Spector, V.E.Zakharov. Physics Lett. A **221** (1996), 73-79.
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[3] G.Iooss, P.I.Plotnikov. Existence of multimodal standing gravity waves, to appear in J.Math. Fluid Mech., 2005.