Semilinear elliptic problems with mixed Dirichlet-Neumann boundary conditions

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This talk contains results in collaboration with prof. I. Peral, and form part of the paper:

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1 Resume of the talk

In this talk we will study semilinear elliptic equations with mixed Dirichlet-Neumann boundary conditions. More precisely, we consider the following problem,

$$(P_\lambda) \equiv \begin{cases} -\Delta u &= \lambda u^q + u^r, \text{ in } \Omega, \\ u &> 0, \text{ in } \Omega, \\ B(u) &= 0, \text{ on } \partial\Omega \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ ($N \geq 3$) is a smooth bounded domain, $1 < r < 2^* - 1 = \frac{N+2}{N-2}$, $0 < q < r$, and the boundary conditions

$$B(u) = u\chi_{\Sigma_1} + \frac{\partial u}{\partial \nu}\chi_{\Sigma_2},$$

where Σ_i, $i = 1, 2$, are smooth $(N-1)$-dimensional submanifolds of $\partial\Omega$ such that $\Sigma_1 \cap \Sigma_2 = \emptyset$, $\overline{\Sigma}_1 \cup \overline{\Sigma}_2 = \partial\Omega$ and $\overline{\Sigma}_1 \cap \overline{\Sigma}_2 = \Gamma$ is a smooth $(N-2)$-dimensional submanifold. We denote by ν the outward unitary normal to the boundary and by χ_{Σ_i} the characteristic function of Σ_i, $i = 1, 2$.
Along the talk I will indicate what are the techniques (of the Dirichlet case) that works and the new techniques we have need to developed to study these kind of problems. Hölder regularity, global multiplicity, uniform L^∞-estimates, nonexistence results, moving of the boundary conditions (in a regular way to be defined)...
To finish, I will point out some related results and problems in progress.
The main bibliography I will use is th following.

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