Bilinear Control Systems: Theory and Applications

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Aim: Bilinear Systems are an important class of nonlinear control systems. The course aims at giving an overview of the main control problems and of some of the mathematical tools (notably differential geometric and Lie algebraic methods) required in the study of bilinear control systems.

Topics:

1. Introductory material
   - manifolds, vector fields, tangent spaces;
   - orbits of vector fields and Frobenius Theorem;
   - controllablity and Chow Theorem;
   - drift versus driftless systems, accessibility versus controllability;
2. Bilinear control systems
   - bilinear systems and matrix transition Lie groups;
   - structure of matrix Lie groups (homogeneous spaces, transitivity, exponential map and canonical coordinates);
   - Lie algebras (Levi decomposition, semisimpicity, solvability, nilpotency, Cartan criteria);
   - controllability properties for bilinear control systems on matrix Lie groups;
3. Control methods
   - feedback linearization;
   - system inversion and differential flatness;
   - feedback stabilization;
4. Applications
   - rigid body motion (rigid bodies on SO(3) and SE(3); system on a sphere);
   - nonholonomic systems (trailer systems, chained form);
   - switching systems (simultaneous stability);
   - quantum control systems (Shrodinger equation, Liouville equation).

References:

There is no book specific for the geometric aspect of bilinear control systems that will be treated in the course. Some parts can be found in

- Murray-Li-Sastry. *A mathematical Introduction to Robotic Manipulation* CRC press 1994. (Controllability; nonholonomic systems; rigid body motion)

- V. Jurdjevic. *Geometric Control Theory*, Cambridge Univ. Press. 1997. (Systems on Lie groups and Homogeneous spaces; classification of Lie algebras)
- D. L. Elliott. *Bilinear Control Systems. Matrices in Action*, Spinger 2009 (broad introduction to bilinear systems)
- D. Liberzon *Switching in Systems and Control*, Birkhauser, 2003