"A quantum field theory with a covariant cutoff in which black hole radiance could be studied"

1. Many ties between gravity and thermodynamics (θD)

For example:

Negative specific heat of globular clusters

Related non-existence of Newtonian equilibrium — cured by GR

Turning point methods for stellar (in)stability

But horizon entropy is doubtless most important tie

2. When we think of ordinary θD , we recognize that it rests on the discreteness of matter

(eg learning $k_{Boltzmann}$ let people count the atoms in a gram of Hydrogen)

(basically k is small because Avogadro's number is big)

But atoms of spacetime $\rightarrow$ discreteness thereof,

and S_{BH} tells their number ($\approx$)

3. parenthetically:

Is BB-radiation an exception?

Well, it's finite thanks to granular structure of light — photons.

Now photons are constituents of light,

but here we want the constituents of spacetime!

Also: Planck spectrum comes from $E = h\nu$, but near horizon the red-shift depresses E, ruining the bound — so on balance I think the original conclusion holds good.

Of course all this is heuristic only.

4. But this raises a question: BH θD is consistent because of BH *radiance.* If discreteness ruins this, the whole picture will crack.

A *naive* cutoff does ruin it. On the other hand we have UV-insensitive reasons to believe in Hawking radiation.

We also have evidence of its robustness against frame-dependent cutoffs.

But better would be a cutoff that didn't spoil local Lor-invar, especially since very large blue-shifts enter the usual derivation.

5. Causal set (causet) provides such a cutoff

$\rightarrow$ one reason to study ϕ on a background causet.

Other reasons:

transplanckian issues in cosmology

understanding how to recover ($\approx$) locality in causal set theory

Latter is *hard* and some residual nonlocality may persist

$\rightarrow$ could be a key signal of discreteness.

6. This lecture: QFT of real scalar field on fixed causet C.

First only the free field, but histories formulation admits interactions.

Many questions for discussion arise here

Criticism and *advice* needed

(probably audience can answer some of these questions)

New light is thrown even on flat space field theory!

7. Outline

- continuum gaussian field theory & the "ground-state condition"

$$\Box - m^2 \to G^{ret} \to \Delta \to W \to \widehat{\phi}$$

Compatibility of CCR with eom

- This does not generalize to causet (compatibility fails)

- Another route to continuum field theory

$$G^{ret} \to \Delta \to W \to \widehat{\phi}$$

- $G^{retarded}$ on causet

- field operators and "ground state" in causet

- DCF (decoherence functional) in causet (just quote results)

- Remarks and questions

8. Minkowski field theory (gaussian)

$$(\Box - m^2)\widehat{\phi} = 0$$

$$(\Box - m^2)G^{ret} = 1 \ , \quad G(x,x') = 0 \text{ unless } x > x' \quad (G \equiv G^{ret})$$

$$\Delta = G - \widetilde{G} = G(x,x') - G(x',x)$$

$$[\widehat{\phi}(x), \widehat{\phi}(x')] = i\Delta(x,x')$$

Observe compatibility:

$(\Box - m^2)\Delta = 0$ since *both* G *and* $\widetilde{G}$ are Green-functions.

(Also observe $\Delta = 0$ if $x \natural x'$.)

These equations furnish an algebra generated by the $\widehat{\phi}(x)$

To represent it we need a "vacuum" or "ground state"

Define vac by

$$\widehat{\phi}^{(+)}|0> = \ 0$$

Then

$$W(x,x') \ = < 0|\,\widehat{\phi}(x)\,\widehat{\phi}(x')\,|0>$$

Observe:

$$W \geq 0 \text{ as a matrix}$$

$$W(x,x') = \sum_k f_k(x) f_k(x')^*$$

where $f_k(x) \propto \exp(ik \cdot x)$

9. Does this generalize to the causal set?

No (not so far) because we lack a compatible pair, $\Box$ and Δ.

We do have a retarded $\Box$ and we could define $G = G^{ret}$ from it,

but Δ would not be a solution then since

$$\Box\,\Delta = \Box\,G - \Box\,\widetilde{G} \neq 1 - 1 = 0$$

10. Another route:

get $W(x,x')$ directly from $G(x,x')$, bypassing eq. of motion!

(not surprising we can do this since G and Δ know about the eom, e.g. $\Box\,[\widehat{\phi},\widehat{\phi}] = 0 \Rightarrow \Box\,\widehat{\phi}$ in center of algebra of operators)

$$W = \text{positive part of } i\Delta$$

i.e.

$$2W = i\Delta + \sqrt{-\Delta^2}$$

Mode expansion of Δ (using L^2-norm) $\Rightarrow$

$$W = R + i\Delta/2$$

where

$$R = \widetilde{R} \text{ is real },$$

$$\Delta = \sum \lambda^2 u \wedge v \ , \quad u \wedge v \equiv u(x)v(x') - v(x)u(x')$$

$$R = \frac{1}{2}\sum \lambda^2 (u \otimes u + v \otimes v)$$

($u \otimes u + v \otimes v$ projects onto span of u and v).

This works in any compact globally hyperbolic (region of) spacetime, and also in $\mathbb{M}^d$.

From W, the whole theory follows by Wick formula (since gaussian assumed)

$$\text{Wick: } \langle \phi_1\phi_2\phi_3\phi_4 \rangle = \langle \phi_1\phi_2 \rangle \langle \phi_3\phi_4 \rangle + \cdots$$

Field operators are

$$\widehat{\phi}(x) = \sum (f(x)\,\widehat{a} + \overline{f(x)}\,\widehat{a}^*)$$

where $f = u + iv$.

11. This carries over to causet *unchanged* once we know $G = G^{retarded}$

$$G = [G^{jk}] \ , \ W = [W^{jk}] \ , \ etc.$$

$$\Delta = G - \widetilde{G}$$

$$2W = i\Delta + \sqrt{-\Delta \cdot \Delta}$$

equivalently

$$W = R + i\Delta/2$$

where

$$\Delta = \sum \lambda^2 u \wedge v \ \Rightarrow \ R = \sum \frac{\lambda^2}{2}(u \otimes u + v \otimes v)$$

and $\widehat{\phi}$ is given by

$$\widehat{\phi} = \sum \lambda^2 (f\ \widehat{a} + f^*\ \widehat{a}^*)$$

where

$$f = \frac{(u - iv)}{\sqrt{2}}$$

Also $\langle \widehat{\phi}\widehat{\phi}\widehat{\phi} \cdots \rangle$ is given by the Wick rule.

REMARK This scheme yields a new type of

"ground state condition": $\overline{W} \cdot W = 0$

12. What is $G^{retarded}$ in a causet?

For a 2D causet and $m^2 = 0$ we have

$$G(x,y) = -1/2\, C(x,y) \ ,$$

where C is the "causal matrix" or "covariant Heaviside function":

$$C(x,y) = 1 \text{ iff } x > y$$

For $\mathbb{M}^2$ and $m^2 \neq 0$, G is a sum over chains:

$$G = \frac{G_0}{1 - m^2 G_0} = G_0 + G_0 m^2 G_0 + G_0 m^2 G_0 m^2 G_0 \cdots$$

For $\mathbb{M}^4$ and $m^2 = 0$, $G \propto L =$ link matrix.

For sprinklings of more general spacetimes, one can try $\underset{K}{\Box}^{-1}$, where $\underset{K}{\Box}$ is one of the retarded d'Alembertians.

Simulations by Johnston and Rideout show good agreement in

the $2D$ diamond (order-interval) when $1 \ll m^{-1} \ll L$

and also for $m = 0$

13. A histories formulation is possible (my original plan for talk!)

- allows for interacting field
- shows that $\phi^{classical}$ obeys an eq. of motion:

$\Box\,\phi = 0$ where $\Box \approx G^{-1}$ which is (almost) *retarded*.

- linear constraints arise on the histories

  e.g. $\phi^j = \phi^k$ if elements j and k are non-Hegelian

  (the theory knows they are not really diffferent!)

- Δ has many small eigenvalues but very few that actually vanish.

$\Rightarrow$ our modes f_α amost span all of field space.

this is very different from the continuum

and begins to answer “Ted’s question”.

14. Questions

* How to set up Hawking radiation computation within above field theory?

We suspect our "ground-state" is H-H, based on analysis and simulations of interval in $\mathbb{M}^2$. What more do we need?

* On "purity" of W

With $\widehat{\phi}$, our $|0>$ is pure by construction ("Fock rep"). But if we start from W, what is the criterion of purity? What is it even for $0+1$ dim: non-rel free particle!

Is GNS purity related to KG version of ground state condition:

$\overline{G} < KG > G = 0.$

What is status of this condition in usual quantum field theory?

Two meanings of "gaussian"? (Wick rule vs $\psi = \exp(-x^2)$)

Do we need purity? If so why?

(This ultimately wants interp of quantum mechanics, cf. coevents!)

* Coherence of theory on overlapping or nested regions

* Related causality issues

NB in classical limit of dcf Schwinger-Lagrangian looks like

$$\widetilde{\varphi} \, \Box \, \phi$$

where $\Box$ is (approx) *retarded*

so equation for *classical field* $\phi = (\phi_1 + \phi_2)/2$ is "causal".

But is *anti*causal equation for $\varphi = \phi_1 - \phi_2$ a bigger worry?

Notice here: our theory should not be perfectly causal, since fixing of causet $\Rightarrow$ conditioning on the future!

similar remark for "coherence question" above (since different regions condition on different things).

This suggests that one can expect full causality and a coherent global theory only in full quantum gravity!