Gravity, Bose-Einstein Condensates and Analogue Models from Theory to Experiment, (also from Kinematics to Thermo-Dynamics ?)

Serena Fagnocchi

Anne McLaren Fellow
School of Physics and Astronomy , Nottingham UK

Gravity as Thermodynamics:
towards the microscopic origin of geometry
SISSA - Trieste, Sept 7 2011

Collaborators:

- R. Balbinot , Bologna University , Italy
- A. Fabbri , Valencia University , Spain
- S, Liberati , S.Finazzi , A. Prain , A. Trombettoni , M. Kormos , SISSA , Trieste , Italy
- E. Copeland's Group , Nottingham , UK
- I. Carusotto , A. Recati , BEC Group , Trento , Italy
- I. Cirac , B. Horstamnn , Max Planck Institut fur Quantenoptik , Munich
- B. Reznik , Tel Aviv University, Israel
- E. Altman , Weizmann Institute , Israel
- E. Demler , Harvard University , US
- P. Krüger , Cold Atoms Group , Nottingham, UK

Outline

1. Analog Models
   - Introduction - Motivations
   - Acoustic Black Holes
   - Cold Atoms
2. Correlations
   - Hawking Radiation
   - Dynamical Casimir Effect
3. From Kinematics to Thermo-Dynamics
   - Dynamics
   - Thermodynamics (speculative)

Quantum Field Theory in Curved Spacetime:

Semiclassical Gravity studies the quantum effects due to the propagation of quantum field in the presence of strong gravitational fields

- Gravity treated classically (Einstein Theory)
- Matter fields are quantized

Important and amazing results:

- Hawking Radiation
- Cosmological particle production
- Super-radiance
- Moving Mirror particle production
- ....

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Yet no experimental observation!

Analog Models of Gravity in Hydrodynamics

. W.G. Unruh, PRL 46 (1981)

- Sound-phonon propagation $\rightarrow$ massless scalar field in curved st
- Hydrodynamical fluids $\rightarrow$ curved space-time
- HOMOGENEOUS SYSTEM $\longrightarrow$ FLAT SPACETIME
- INHOMOGENEITIES $\longrightarrow$ CURVED SPACETIME
- Tool to investigate effects otherwise NOT accessible

ANALOGY WITH HYDRODYNAMICS

- Continuity and Bernoulli Eqs.
  for irrotational, inviscid fluid
  n= density, $\vec{v} = \vec{\nabla}\theta$= flow velocity, $\mu(n)$= specific enthalpy

$$\dot{n} + \vec{\nabla} \cdot (n\vec{v}) = 0 \qquad \dot{\theta} + \frac{1}{2}v^2 + \mu(n) = 0$$

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- Fluctuations on top of mean field solution: $n + n_1\,, \theta + \theta_1$
  Linearized Eqs:

$$\dot{n}_1 + \vec{\nabla} \cdot (n\vec{v}_1 + n_1\vec{v}) = 0 \qquad \dot{\theta}_1 + \vec{v} \cdot \vec{v}_1 + \frac{c^2}{n} n_1 = 0$$

with the sound velocity: $c^2 = n d\mu/dn$.

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  with the sound velocity: $c^2 = nd\mu/dn$.
- Putting n_1 into the first eq....

Analog Models

....one gets:

$$\left\{-\partial_t\left[\frac{n}{2c^2}(\partial_t+\vec{v}\cdot\vec{\nabla})\right]+\vec{\nabla}\cdot\left[\frac{\vec{v}n}{c^2}(\partial_t+\vec{v}\cdot\vec{\nabla})+n\vec{v}\cdot\vec{\nabla}\right]\right\}\theta_1=0$$

$$\longrightarrow \quad \Box\theta_1=\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\right)\theta_1=0$$

$\Box$: D'Alembertian in curved space
described by the "acoustic metric" $g_{\mu\nu}$:

$$g_{\mu\nu}\equiv\frac{n}{mc}\begin{pmatrix}-(c^2-v^2) & -\vec{v}^T\\ -\vec{v} & \mathbf{1}\end{pmatrix}$$

Analog Models

Core of the Analogy: $\Box\theta_1 = 0$

Acoustic metric:

$$g_{\mu\nu} \equiv \frac{n}{mc} \begin{pmatrix} -(c^2 - v^2) & -\vec{v}^T \\ -\vec{v} & \mathbf{1} \end{pmatrix}$$

- Sound propagates along null geodesics of $g_{\mu\nu}$.
- Geometrical analogy
- θ_1 massless scalar field propagating on curved spacetime with c, v, n functions of $(t, \vec{x})$.
- Choosing different space-time profiles for $c, v, n \rightarrow$ different metrics
- For $v = c \rightarrow g_{\mu\nu}$ black hole metric.

ANALOG MODELS OF GRAVITY IN CONDENSED MATTER

POWERFUL TOOL TO THEORETICALLY AND EXPERIMENTALLY INVESTIGATE QFT IN CURVED (also flat) SPACES PROBLEMS

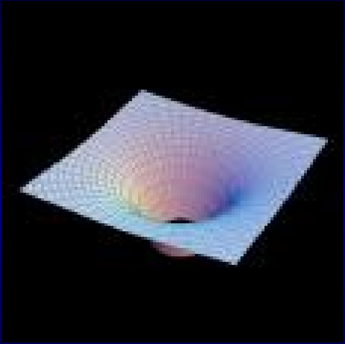
EXPERIMENTAL:

- Black Holes evaporation: Hawking Effect
- Cosmological expansion
- Dynamical Casimir effect
- ...

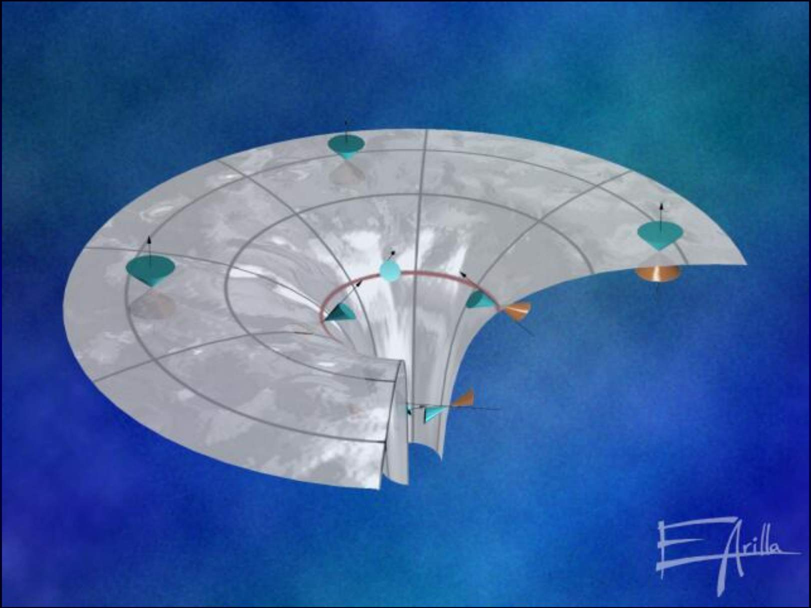
THEORETICAL:

- Trans-Planckian problem: Effects of modified dispersion relations
- Extreme black holes
- Black Holes thermodynamics (?)

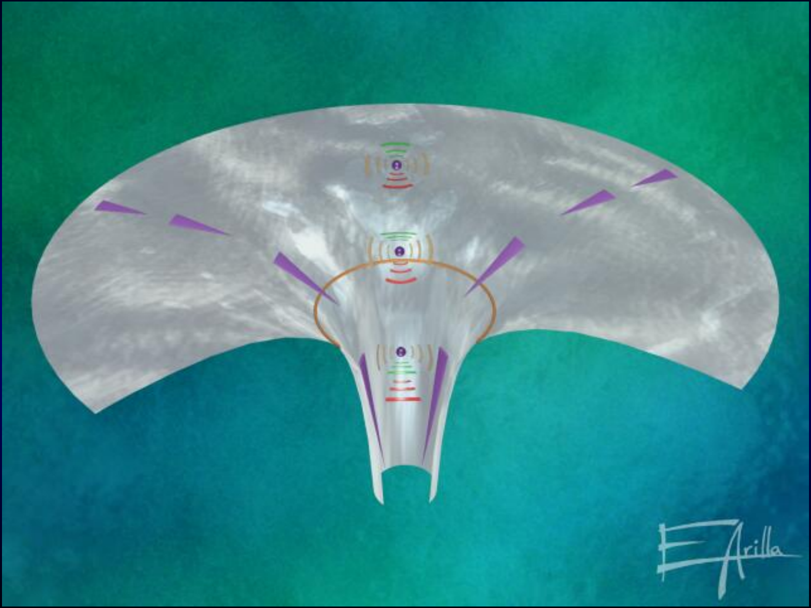
GRAVITATIONAL BLACK HOLES



- Geometrical Objects
- Curvature so high that even light cannot escape
  Trapped regions



What is an acoustic Black Hole ?



Hawking Radiation

. S.W. Hawking, Nature 248 (1974)

Semiclassically, black holes are not "black" objects, but radiate particles after the horizon formation.

Thermal flux of particles detected asymptotically far from a black hole.

- Quantum effect
- Stationary emission
- Thermal spectrum
- Pure geometrical effect $\rightarrow$ independent on dynamics
- Still unobserved: 10^{-8} K (CMB: ~ 3 K)

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- Pure geometrical effect $\rightarrow$ independent on dynamics
- Still unobserved: 10^{-8} K (CMB: ~ 3 K)
- Since the analogy with fluids is both classical and quantum, in acoustic black holes Hawking radiation is expected as a thermal phonons emission

Acoustic Black Holes

Possible candidates:

- Atomic Bose-Einstein condensates
  . Garay, Anglin, Cirac, Zoller, PRL 85(2000)
- Quasi-particle excitations in superfluid Helium
  . Jacobson, Volovik, PRD 58 (1998)
- Fermi gases . Giovanazzi, PRL 94 (2005)
- Slow-light . Leonhardt, Piwnicki, PRL 85(2000)
- Nonlinear electromagnetic waveguides
  . Schutzhold, Unruh, PRL 95 (2005)
  . Philbin et al., Science 319 (2008)
- Ion rings . Horstmann, Reznik, SF, Cirac, PRL 104 (2010)
- Laser pulse filaments . Belgiorno et al. NJP 12 (2010)

Ultra-Cold Atoms : Bose Einstein Condensates
probably the most promising candidates to observe gravitational quantum effects:

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Bose Einstein condensates:

- Pure quantum systems
- Ultra-cold temperature ($<$ 100 nK)
  $\rightarrow$ Hawking temperature $\sim$ 10 nK
- Huge technological improvement
- Hydrodynamical description

- Technical details: Bose-Einstein condensates

are described by the GPE for a classical scalar field ψ :

$$i\hbar\partial_t\psi(t,x) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{ext}(x) + gn(x)\right)\psi(t,x) \qquad (1)$$

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with n: density, θ: phase , v: gradient of θ , g: interaction strength

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- $\partial_t n + \nabla\cdot(nv) = 0$
- $m\partial_t v + \nabla\left(\frac{mv^2}{2} + V_{ext} + gn - \frac{\hbar^2\nabla^2\sqrt{n}}{2m\sqrt{n}}\right) = 0$

Gross-Pitaevskii Equation

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Again linearizing : $n + \hat{n}_1$ and $\theta + \hat{\theta}_1$
$\hat{n}_1, \hat{\theta}_1$: quantum fluctuations,

- Technical details: The Analogy in BEC

Bogoliubov-De Gennes equations in hydrodynamical approximation:

$$\Box\hat{\theta}_1 = 0 \qquad\qquad \hat{n}_1 = -g^{-1}(\partial_t\hat{\theta}_1 + \vec{v}\cdot\nabla\hat{\theta}_1)$$

$\Box$: D'Alembertian in curved space
described by the "acoustic metric" $g_{\mu\nu}$:

$$g_{\mu\nu} \equiv \frac{n}{mc}\begin{pmatrix} -(c^2 - v^2) & -\vec{v}^T \\ -\vec{v} & \mathbf{1} \end{pmatrix}$$

For acoustic BH: $T_H = \frac{\hbar k}{2\pi c K_b}$
with $k = \frac{1}{2}\frac{d}{dx}(c^2 - v^2)|_H$ surface gravity on H

Hawking Radiation's direct detection
is still very difficult

In BEC: $T_{Hawking} \sim$ few nK
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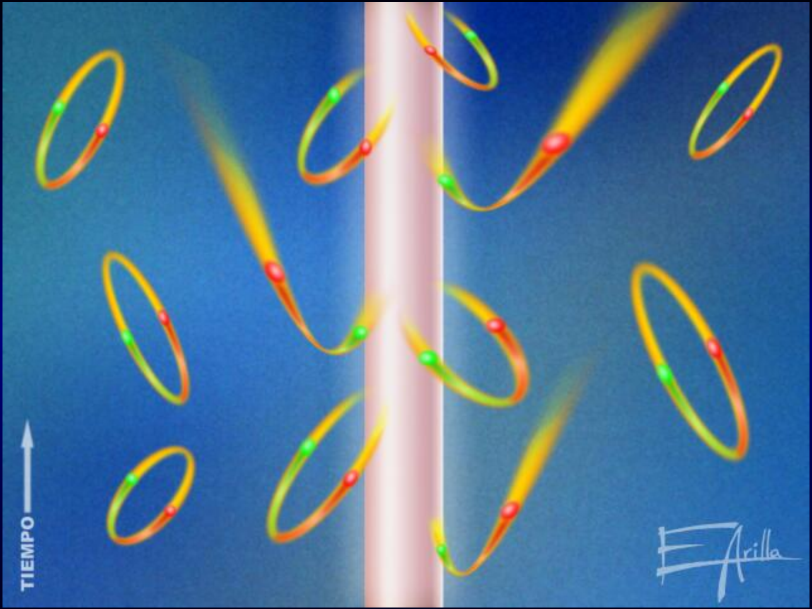
- Go to Non-local Correlations



PARTICLES PRODUCED IN PAIRS !!

Non-local Density Correlations

Acoustic Black Hole using supersonic BEC

Diluted BEC: n (density) and θ (phase) solution of the GP eqs.

Fluctuations: $n + \hat{n}_1$; $\theta + \hat{\theta}_1$

- $G_2(x, x') = \langle \hat{n}(x)\hat{n}(x')\rangle - \langle \hat{n}(x)\rangle\langle \hat{n}(x')\rangle = \langle \hat{n}_1(x)\hat{n}_1(x')\rangle$
  $\rightarrow$ x and x' on opposite sides wrt the horizon

FIND $\langle\hat{\theta}_1(x)\hat{\theta}_1(x')\rangle$!

Correlations

- Technical details:
  QFT in curved space - Black Holes

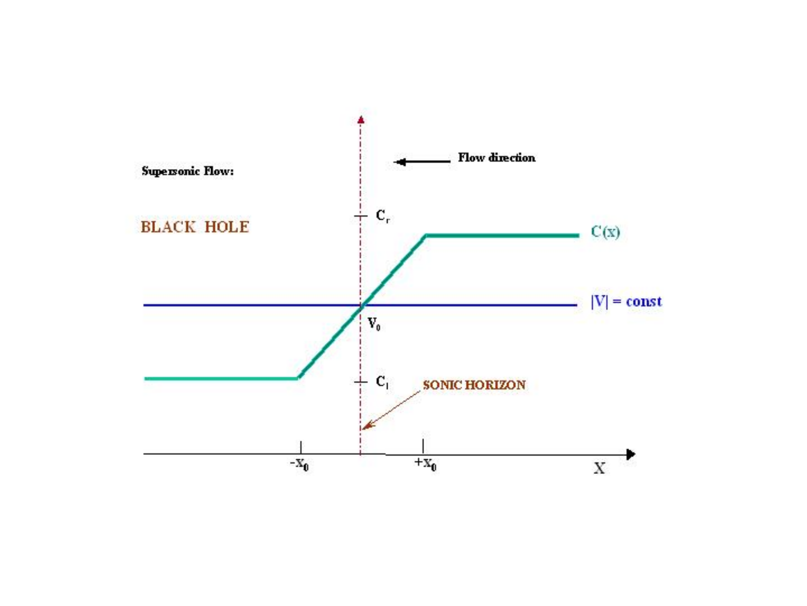
In 1D: $\langle \hat{\theta}_1(x)\hat{\theta}_1(x')\rangle_U \propto \ln\left[\Delta X^-(x,x')\Delta X^+(x,x')\right]$

- $\Delta X^-, \Delta X^+$: light-cone distance
- Motion of downstream modes unaffected: $X^+ = ct + x$
- Distortion of upstream propagating modes is Universal:
  $X^- = \pm 1/k e^{-kx^-}$
- Surface gravity on the sonic horizon: $k = \frac{1}{2}\frac{d}{dx}(c^2 - v^2)|_H$

- Unruh, PRL 46 (1981)
- Visser, CQG 15 (1998)
- Balbinot, SF, Fabbri, Procopio, PRL 94 (2005)

- Balbinot, SF, Fabbri, PRD 71 (2005)

BEC Setup Proposal



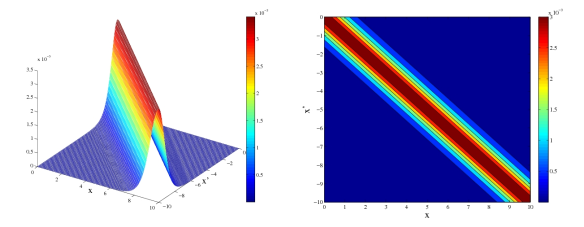
Non-Local IN/OUT Density Correlations

$$\langle \hat{n}_1(x)\hat{n}_1(x')\rangle = \frac{\hbar^2}{16\pi g_1 g_2}\frac{1}{n\sqrt{\xi_1\xi_2}}\frac{c_1 c_2}{(c_1 - v_0)(c_2 - v_0)}$$

$$\times \frac{k^2}{\cosh^2\left[\frac{k}{2}\left(\frac{x}{c_1 - v_0} - \frac{x'}{c_2 - v_0}\right)\right]} + O(x - x')^{-2}.$$

Non-Local IN/OUT Density Correlations

$$\langle \hat{n}_1(x)\hat{n}_1(x')\rangle \propto \frac{T^2_{Hawking}}{\cosh^2(k(x+x')/2v_0)}$$



Peak for $x' = -x$!!

The Hawking and the partner particles are exactly opposite with respect to the horizon

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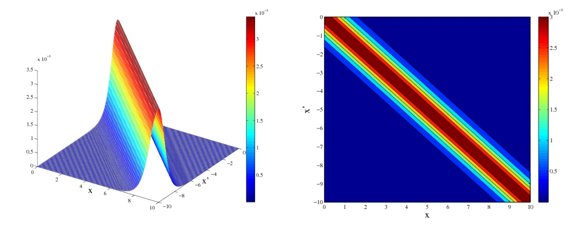

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The Hawking and the partner particles are exactly opposite with respect to the horizon

No Black Hole: $\langle \hat{n}_1(x)\hat{n}_1(x')\rangle \propto (x-x')^{-2}$

Approximated analytical result confirmed by

TIME DEPENDENT FORMATION OF AN ACOUSTIC HORIZON

FULL MICROSCOPIC MANY-BODY PHYSICS TAKEN INTO ACCOUNT IN A BOGOLIUBOV-LIKE NUMERICAL SIMULATION

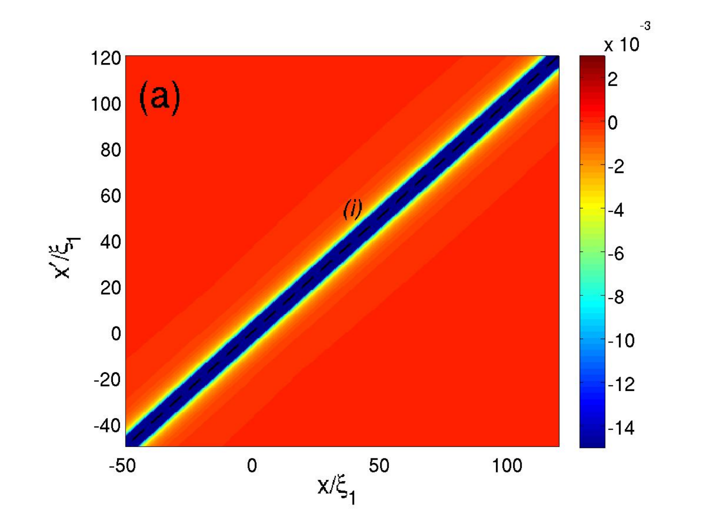
Full dispersion relation (quadratic at high frequency), beyond hydrodynamical approximation, finite Temperature, time-evolution (backreaction)...

- Carusotto, Fagnocchi, Recati, Balbinot, Fabbri, New J. Phys. 10 (2008)

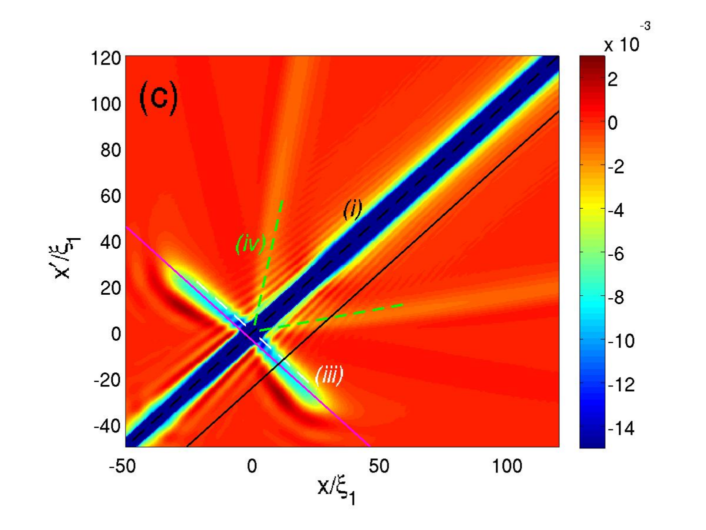
Semi-analytical analysis:

- Recati, Pavloff, Carusotto, PRA 80 (2009)

Numerical simulation - Initial: $G_2(x, x')/n^2$



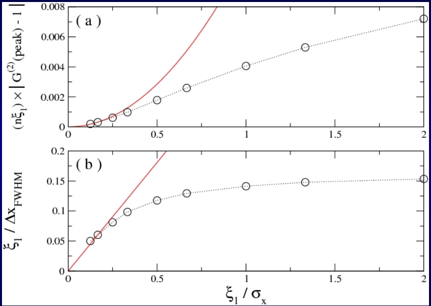
Numerical simulation Results: $G_2(x, x')/n^2$



System parameters: $v_0/c_1 = 0.75$, $v_0/c_2 = 1.5$,
H spatial width$= \sigma/\xi_1 = 0.5$, $T_0 = 0$

Numerical simulation Results

Comparison with the analytical predictions:



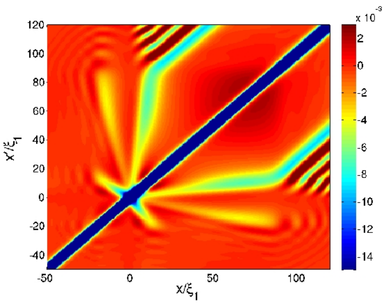
Excellent agreement in the hydrodynamical limit.

Still Hawking effect robust even beyond

Numerical simulation Results

Role of thermal fluctuations:

- Numerical simulation with $T = 0.1\mu \gg T_{Hawking} \sim 10^{-2}\mu$



Correlations in acoustic Black Holes

Realizing any concrete experiment to observe Hawking radiation still is a non trivial task.

Interest on this kind of experiments is exponentially increasing

Acoustic black hole:

- Lahav et al, PRL 105 (2010)

Stimulated Hawking emission:

- Weinfurtner et al, PRL 106 (2011)

Controversial result:

- Belgiorno et al, NJP 12 (2010)

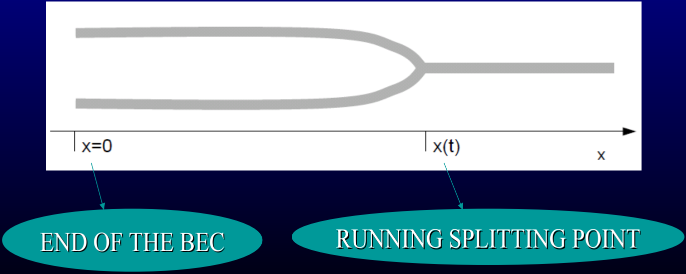
Dynamical Casimir effect - BEC Zipper

- **Casimir Effect**:
  Two plates in vacuum attract each other due to em vacuum. V
- **Dynamical Casimir**:
  Plates in non uniformly accelerated motion $\rightarrow$ real photons are produced. Not V
- Originally DCE is a Moving Mirrors problem
  . Moore, JMP 11 (1970)
  . Carlitz, Willey, PRD 36 (1987)
  Then huge leterature

The Moving mirror problem : THE BEC ZIPPER

. SF, Altman, Demler, to appear (hopefully)

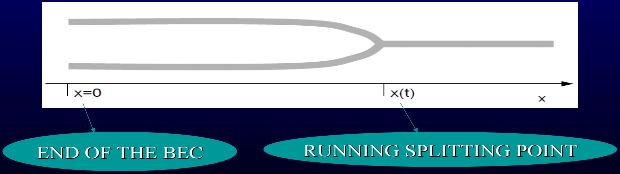
UNZIPPING A 1-D BEC through a Y-junction.
Dynamics of the phase difference φ between the two arms $\rightarrow$ well known QFT description. (. A. Polkovnikov et al, PNAS (2006))



THE BEC ZIPPER– QFT mapping :

- $1D$-problem for the phase difference $\varphi = \varphi_1 - \varphi_2$:
  $S = -\frac{K}{2}\int dt\, dx \left[(\partial_t \hat{\varphi})^2 - \frac{1}{c^2}(\partial_x \varphi)^2\right]$
- Eq. of Motion for φ:
  $(c^2\partial_t^2 - \partial_x^2)\varphi = 0$
- with two boundary conditions:
  $\varphi|_{x_m(t)} = 0 \rightarrow$ at the spitting point
  $\partial_x\varphi|_{x=0} = 0 \rightarrow$ at the edge (to avoid current)

Where c is sound speed !



THE BEC ZIPPER

- Natural realization of Dynamical Casimir effect in an ultra-cold atom interferometer
- Emitted radiation can be directly measured by the interference fringes
- It can be experimentally implemented to give the first (maybe second) experimental evidence of this important prediction

  . Wilson et al, arXiv: 1105.4714, DCE fron superconducting circuits

  . Westbrook, DCE from BEC, unpublished
- Outcomes will depend just on the trajectory of the splitting point
  i.e. exponential traj.$\rightarrow$thermal emission
  (link to Hawking radiation). Possibility to engineer splitting traj. to get new physics

Conclusion First Part

- Importance of Analog Models to test undetectable effects and to suggest new physics
- Importance of Correlation measurement for testing far from equilibrium dynamics
- Major issues solved before real experiments to detect Hawking radiation
- The BEC UNZIPPING is a very interesting non equilibrium process mapping in a well known QFT problem: The Dynamical Casimir Effect

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and now...Second Part!

DYNAMICS

So far only Kinematics. How to go to Dynamics?
EoM are different!

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EoM are different!

- BACKREACTION
  of Hawking emission in acoustic black holes :
  . Balbinot et al, PRL 94, PRD 71 (2005), PRA 75 (2007)
  . Schützhold et al, PRD 76 (2007)
- Some gravitational EoM mapped into fluid-dynamics equations under some conditions
  . Cadoni, Mignemi, PRD 72 (2005)
- AdS/CFT correspondence : mapping conformally invariant problems (like phase transitions) into
  AdS_5+BH+Charge+Spin problem
  . Sadchev (2011)

THERMODYNAMICS ? speculative...

And now how step into thermodynamics ?

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ENTROPY of acoustic Black Holes

- Is it possible to associate an entropy to acoustic BH?

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Some considerations:

- Area comes directly from Einstein eqs, but not only in gravity we find entropy scaling as surfaces (many examples in atomic phys.)
- Entropy of a scalar field in flat-BH spacetime (+cutoff) in the vacuum state restricted on a close surface scales as the surface area . Srednicki, PRL 71 (1993)
  . Bombelli et al, PRD 34 (1986)

But above all.....

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BHO!
It's worth investigating...
knowing more about entropy in acoustic BH can help the understanding of gravity

- What does the BH entropy come from? What is it associated with?
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