Entropy of causal horizons from causal sets

A proposal for a microscopic account of horizon entropy

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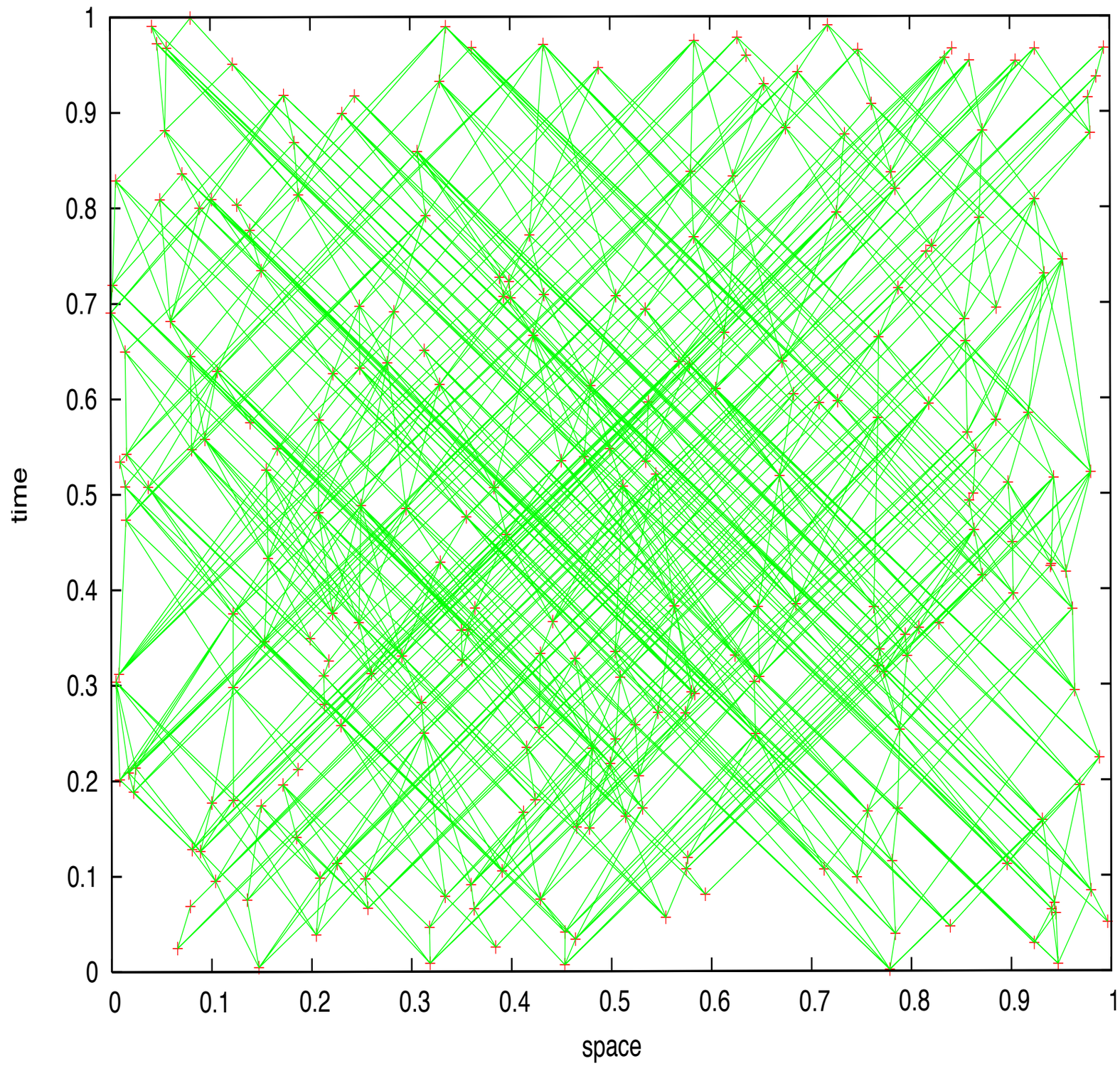
Seeking the statistical mechanics of horizon thermodynamics

I will assume:

- Black hole thermodynamics is due to the causal nature of the horizon. Indeed, **all** causal horizons obey the laws of thermodynamics (Jacobson & Parentani)
- Without a physical cutoff the entropy of a black hole would be infinite, and the finite physical value of black hole entropy tells us the cutoff scale (Sorkin)

A simple strategy: seek a universal microscopic explanation of the entropy of causal horizons based on a description of spacetime that is atomic at the Planck scale and in which causal order is meaningful. It turns out that any (causal) Lorentzian spacetime can be modelled by **nothing more** than a discrete (causal) order or causal set (Bombelli, Lee, Meyer, Sorkin).

For Example



Future

Past

This is what Minkowski spacetime is (like)

Notes

- For every (causal) Lorentzian spacetime, (M,g) there are many discrete orders to which M is a continuum approximation, gotten (mathematically, not physically) by "sprinkling" into M at density $\rho = l^{-d}$ where l is a fundamental length of order the Planck length.

- Everything I will say in this talk pertains to "averages over sprinklings" (fluctuations are a whole other, important story, involving introduction of a "mesoscale"-- a feature of many (all?) quantum gravity approaches)

- In a causal set, C, a causal horizon can be identified with a **partition** of the causal set into the past of a future inextendible chain and the complement of this past set.

- We'll assume that the hypersurface Sigma, which intersects the horizon and on which we want to evaluate the entropy is similarly identifiable with a partition of C into the past and future of Sigma.

What does horizon entropy count?

(**Kinematics only**: "the entropy of a gas is the number of molecules")

- Old idea of Dou and Sorkin: a spacetime with a horizon is actually a causal set and the horizon entropy counts links straddling the horizon: count the number of links across H in the vicinity of Sigma and hope to get on average, in the limit $l \to 0$,for all horizons in dim d

$$\langle N_{\text{links}} \rangle \to c_d \, \frac{A_d}{l^{d-2}}$$

- A new proposal (Benincasa, FD, Rideout) uses the causal set action but may turn out to be a realisation of the Dou-Sorkin link counting (a way of eliminating the overcounting that the nonlocality of causal sets produces)

Causal set actions

(Benincasa & FD)

$$\frac{\mathcal{S}^{(2)}(\mathcal{C})}{\hbar} := \zeta^{(2)}\left(\frac{1}{2}N - N_1 + 2N_2 - N_3\right)$$

$$\frac{\mathcal{S}^{(4)}(\mathcal{C})}{\hbar} := \zeta^{(4)}\left(N - N_1 + 9N_2 - 16N_3 + 8N_4\right)$$

$\zeta^{(d)}$ is a positive dimensionless constant of order one. In 4d it is proportional to $\left(\frac{l}{l_p}\right)^2$

- There are causal set actions in all other dimensions (Benincasa, FD, Lisa Glaser)
- We have reason to believe that the average over sprinklings into (M,g) is related to the continuum action, for example:

$$\left\langle \mathcal{S}^{(4)} \right\rangle \approx \frac{1}{2l_p^2}\int_M d^4x\sqrt{-g}\,R(x) \;+\; \text{boundary terms depending on } l$$

For an interval in 2d <S> = constant, and in 4d <S> goes like constant x l^{-2}

Nonlocality

The causal set action is nonlocal. Let $\mathcal{C} = X \cup Y$ be a partition of $\mathcal{C}$ .Then

$$\mathcal{S}(\mathcal{C}) \neq \mathcal{S}(X) + \mathcal{S}(Y)$$

Define

$$\mathcal{I}(X,Y) := \frac{1}{\hbar\zeta}\Big(\mathcal{S}(X) + \mathcal{S}(Y) - \mathcal{S}(X \cup Y)\Big)$$

Comparing with the mutual information

$$I(A,B) = H(A) + H(B) - H(A \cup B)$$

we call $\mathcal{I}(X,Y)$ the "Spacetime Mutual Information" (SMI)

Proposal

Let H be the horizon and Σ be a (null) hypersurface intersecting H. Let M be the spacetime to the past of Σ. Then $M = X \cup Y$ where $X = M \cap J^+(H)$ and $Y = M \cap J^-(H)$

$$S_{H,\Sigma} = \mathcal{I}(X, Y)$$

For example:

$$\mathcal{I}^{(2)}(X, Y) = N_1(X, Y) - 2N_2(X, Y) + N_3(X, Y)$$

Does it "work"?

By this I mean, does it give, on average, for all causal horizons and in the limit of large horizon area:

$$c_{(d)} \frac{A}{l^{d-2}}$$

with $c_{(d)}$ of order one.

The evidence so far is promising but far from convincing:

- In 2d, we can calculate <S> analytically for Rindler and the result is 1/2.
- Numerical simulations for 2d deSitter horizons are consistent with 1/2.
- We're working on 2d Schwarzschild.
- We're also working on 3d Rindler and 3d Schwarzschild.
- The 3d Rindler result is consistent with $c_{(3)} \frac{L}{l}$ where the coefficient is of order one.

Notes

- It may not work, in which case it's back to the drawing board. However, if it does work it would be **universal** and arises because of the dichotomy of spacetime by a causal horizon and because all horizons are locally like Rindler.

- Relation to Padmanabhan's observation that the Grav. b.t. on horizon = entropy?

- Would be non-equilibrium. So a Generalised Second Law (GSL) is already (essentially) proved for it at the semiclassical level (Sorkin). However, we would still need to prove that (up to some order one factor) it actually deserves the title of an entropy by proving a GSL from the microscopic, quantal theory.

- Good news: the spacetime mutual information is defined in causal terms which is necessary to fit into Sorkin's "proof scheme" for the GSL in full quantum gravity.

- Bad news: the SMI is a fully spacetime (not spatial) quantity and we (currently) do not have an spacetime understanding of entropy.

- Good news: quantum causal sets require a spacetime (aka sum-over-histories) framework anyway.

- Being optimistic: trying to prove a GSL using this quantity will lead us to a spacetime ("histories") quantum mechanical understanding of entropy.

Final Comment

Jacobson and Parentani stated, “One of the key questions is whether the local notion of horizon entropy density is a valid concept, or whether something essentially global is involved.” The proposal here is that horizon entropy is global in the sense of pertaining to a causally defined dichotomy of spacetime and yet (if it works) it is effectively localized.

The answer to Jacobson and Parentani's key question would then be, “Yes and yes.”