Clusters, clustering, and cosmology

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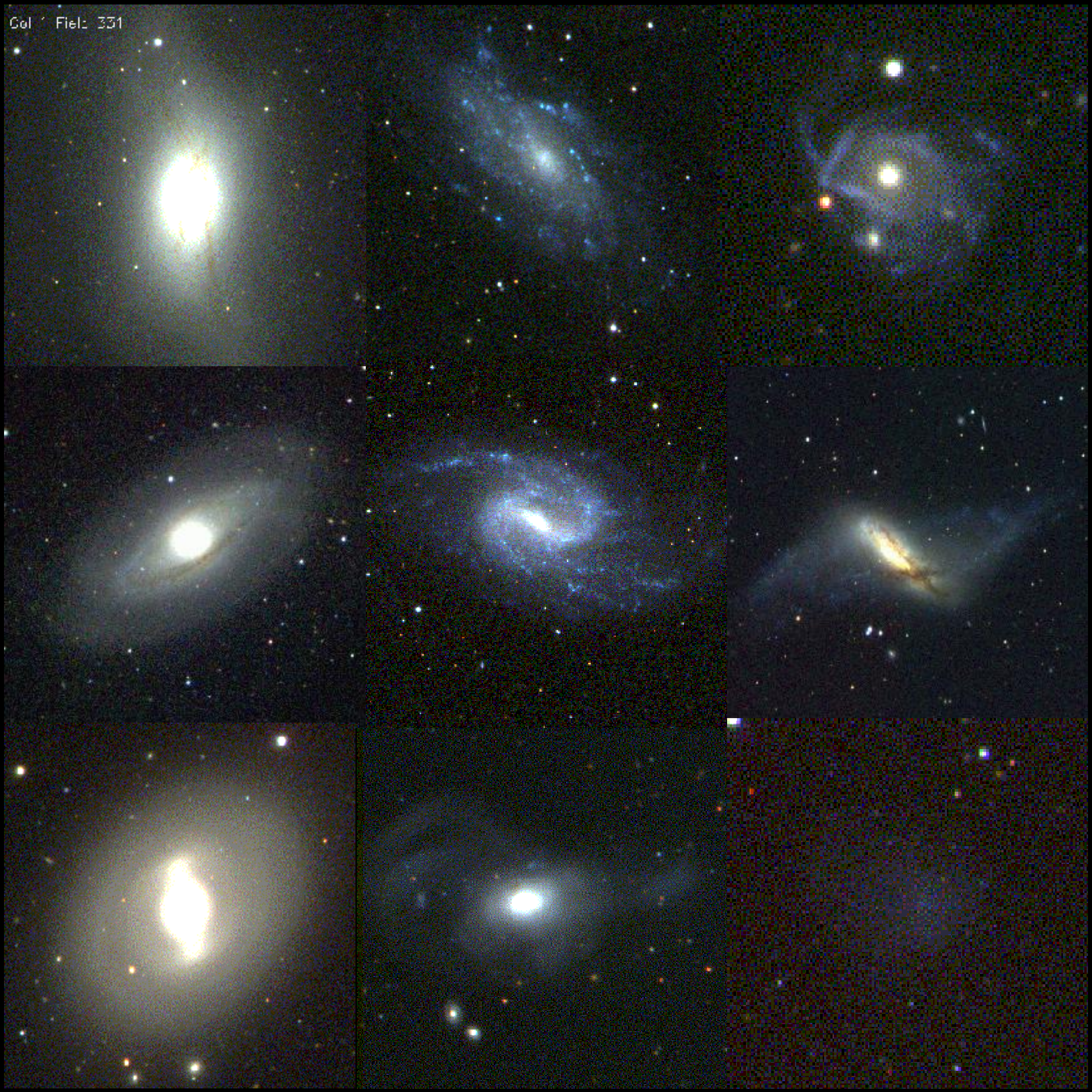
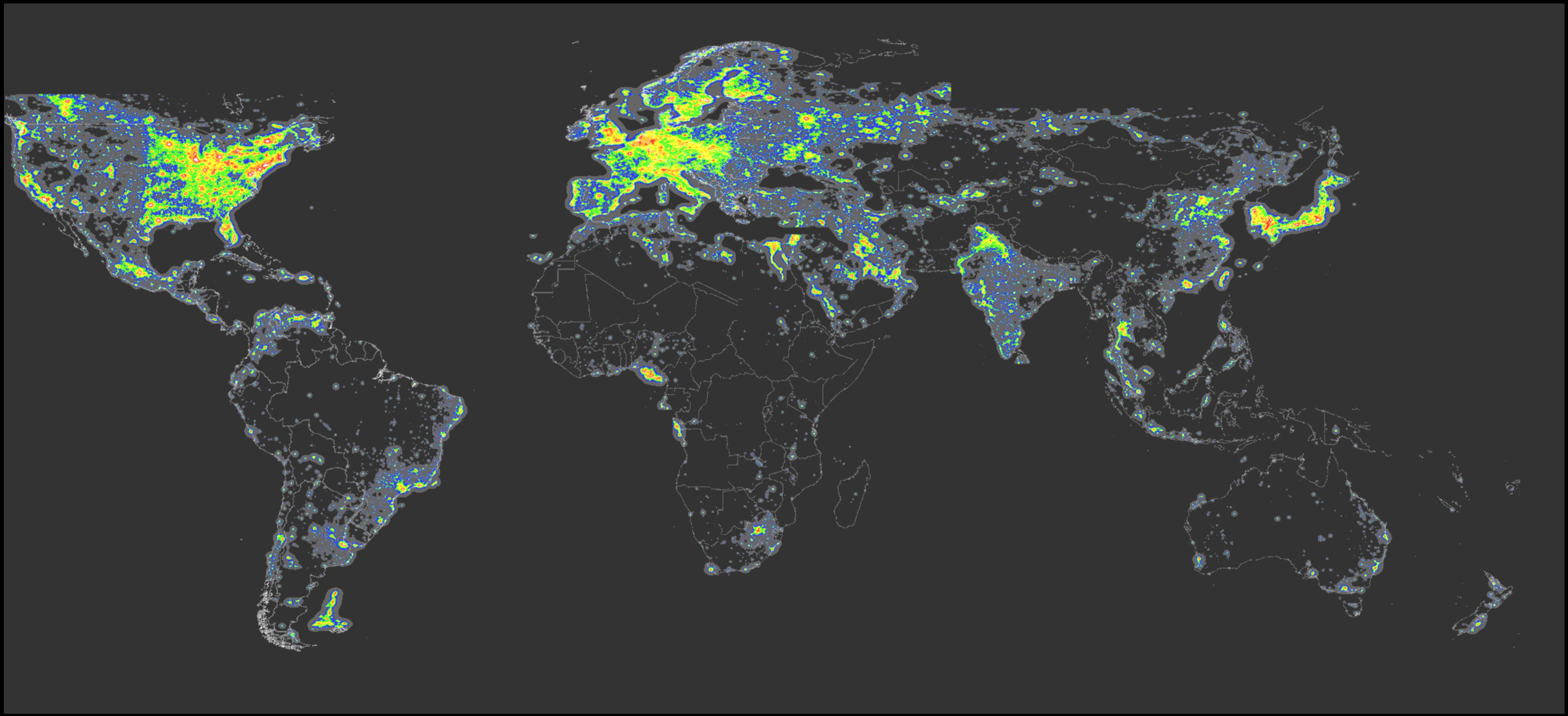
- The Halo Model
  - Observations/Motivation
  - Weights
- Halo abundances
  - The ansatz
  - Correlated steps and large scale bias
  - Ensemble averages: Halos and peaks
  - Modified gravity

Galaxy spatial distribution depends on galaxy type (luminosity, color, etc.)

Not all galaxies can be unbiased tracers of the underlying Mass distribution



Light is a biased tracer



Understanding bias important for understanding mass

How to describe different point processes which are all built from the same underlying distribution?

THE HALO MODEL

A THEORY OF THE SPATIAL DISTRIBUTION OF GALAXIES*

J. NEYMAN AND E. L. SCOTT
Statistical Laboratory, University of California
Received February 18, 1952

ABSTRACT

A theory of the spatial distribution of galaxies is built, based on the following four main assumptions: (i) galaxies occur only in clusters; (ii) the number of galaxies varies from cluster to cluster, subject to a probabilistic law; (iii) the distribution of galaxies within a cluster is also subject to a probabilistic law; and (iv) the distribution of cluster centers in space is subject to a probabilistic law described as quasi-uniform. The main result obtained is the joint probability generating function $G_{N_1, N_2}(t_1, t_2)$ of numbers N_1 and N_2 of galaxies visible on photographs from two arbitrarily placed regions ω_1 and ω_2, taken with fixed limiting magnitudes m_1 and m_2, respectively. The theory ignores the possibility of light-absorbing clouds. The function $G_{N_1, N_2}(t_1, t_3)$ is expressed in terms of four functions left unspecified, which govern the details of the structure contemplated. Methods are indicated whereby approximations to these functions can be obtained and whereby the general validity of the hypotheses can be tested.

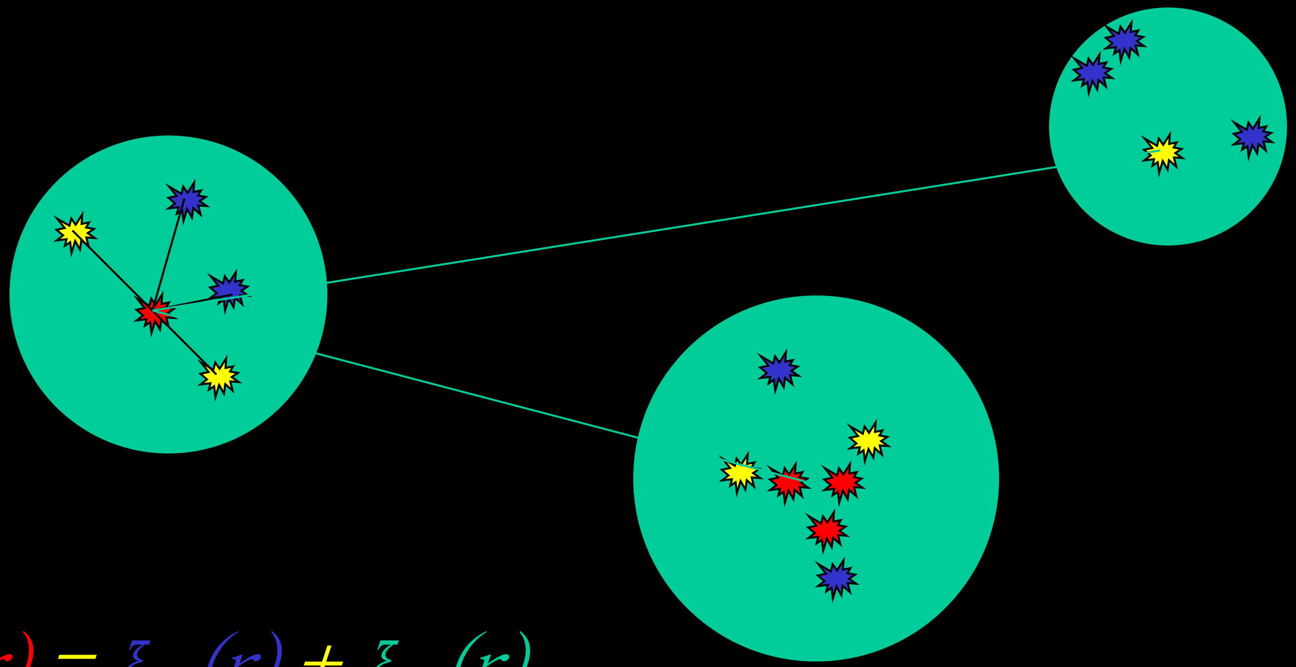
Center-satellite process requires knowledge of how

1) halo abundance; 2) halo clustering; 3) halo profiles;
4) number of galaxies per halo; all depend on halo mass.

(Revived, then discarded in 1970s by Peebles, McClelland & Silk)

The halo-model of clustering

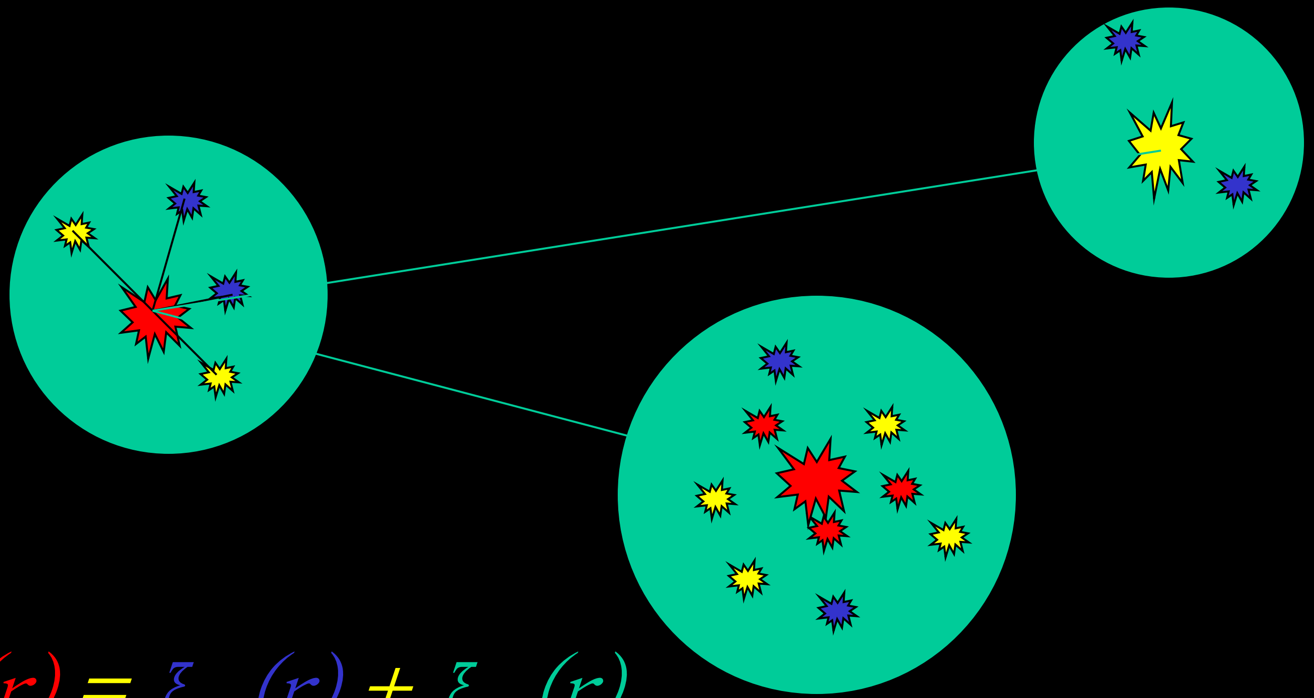
- Two types of pairs: both particles in same halo, or particles in different halos



- $\xi_{obs}(r) = \xi_{1h}(r) + \xi_{2h}(r)$
- *All* physics can be decomposed similarly: ‘nonlinear’ effects from within halo, ‘linear’ from outside

The halo-model of clustering

- Two types of particles: central + 'satellite'



- $\xi_{obs}(r) = \xi_{1h}(r) + \xi_{2h}(r)$
- $\xi_{1h}(r) = \xi_{cs}(r) + \xi_{ss}(r)$

To implement
Neyman-Scott program
need model for
cluster abundance
and cluster clustering

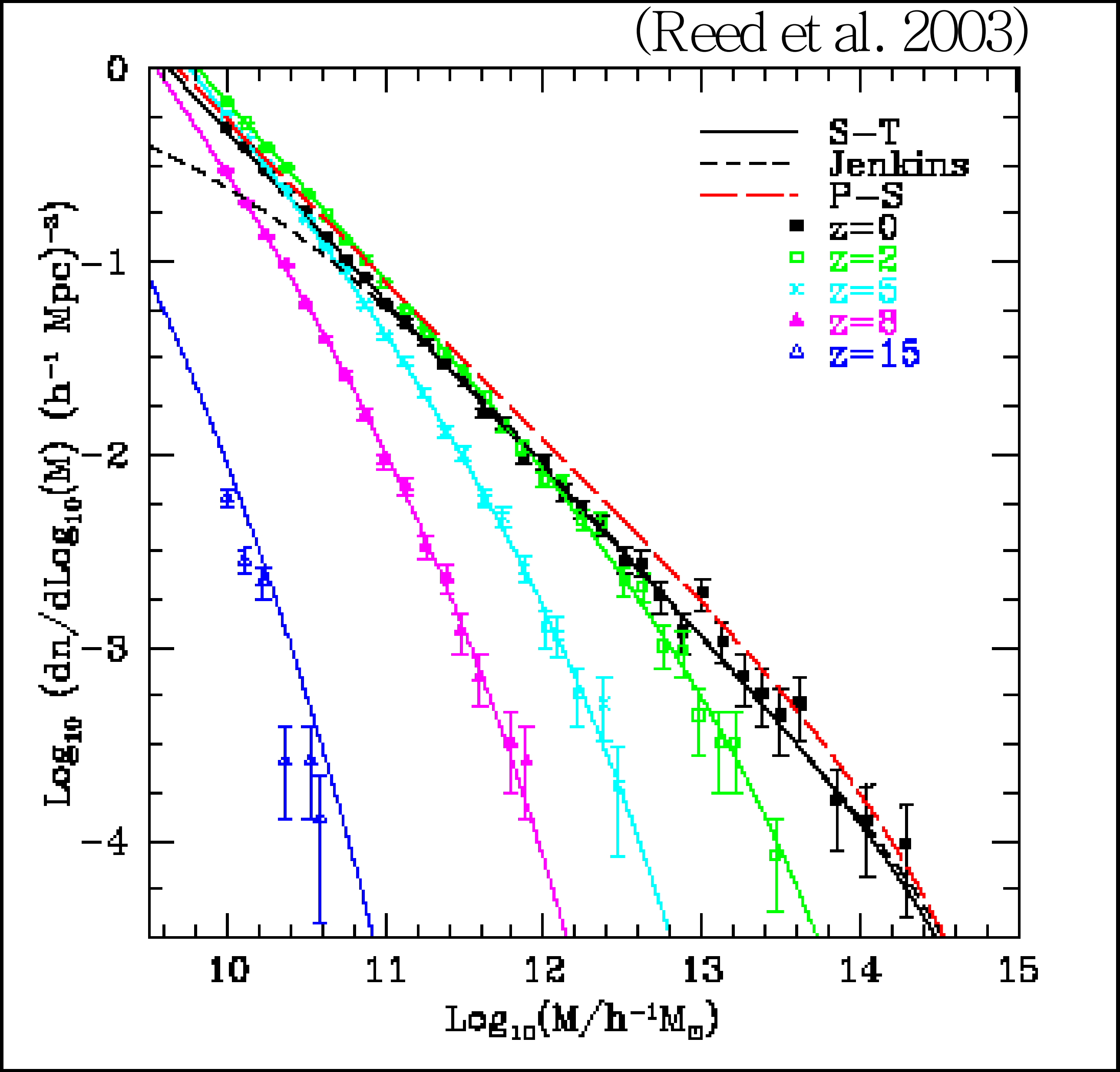
Next generation of surveys will
have 10^9 galaxies in 10^5 clusters

Bells and whistles

- Scatter in mass-concentration relation
  - Different profiles for red vs blue
- Distribution of halo shapes
  - Correlation of shapes with surrounding large scale structure
- Substructure: correlation with concentration/formation time/environment?
  - Correlation of substructure with large scale structure

The Halo Mass Function

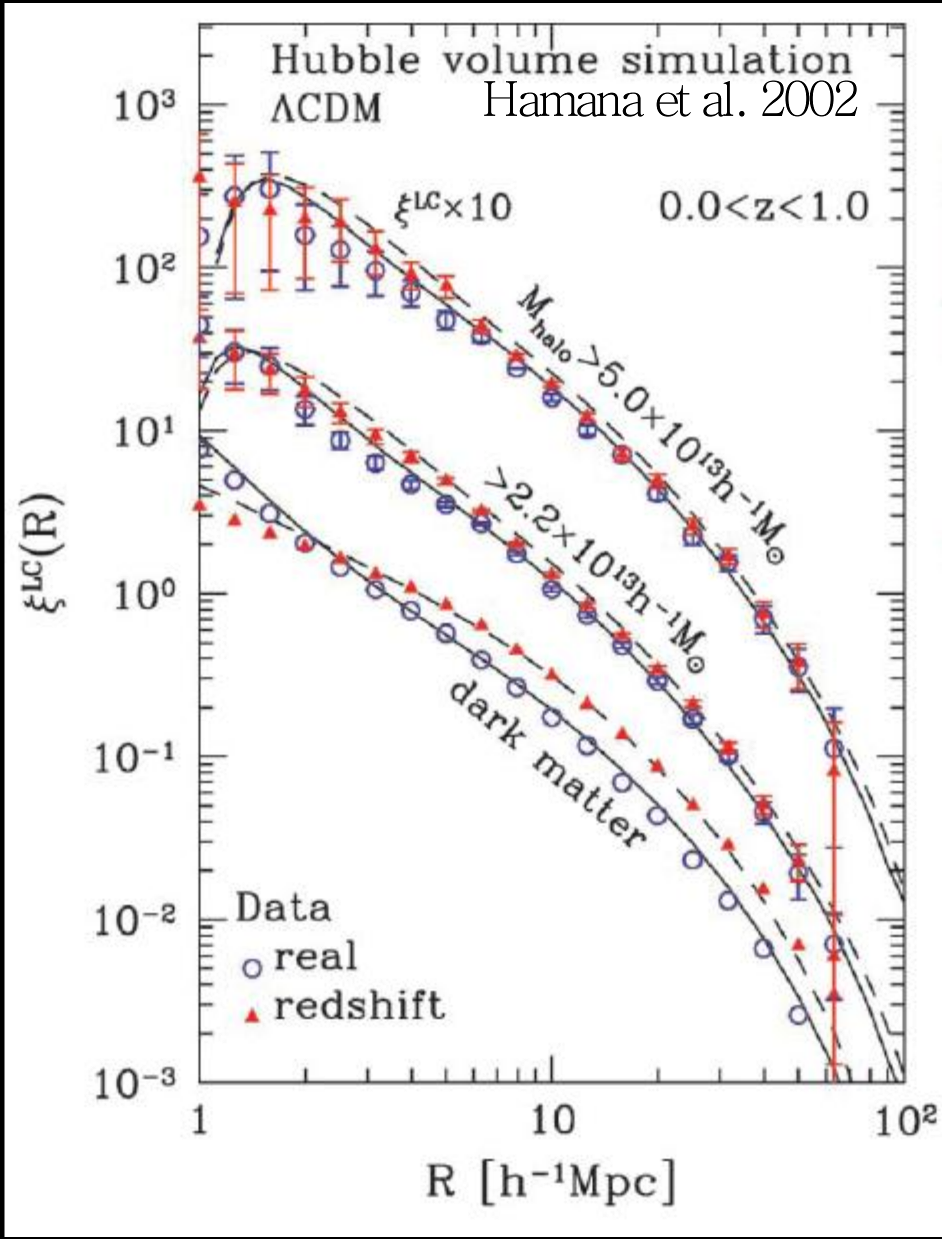
- Small halos collapse/virialize first
- Can also model halo spatial distribution
- Massive halos more strongly clustered



(current parametrizations by Sheth & Tormen 1999; Jenkins etal. 2001)

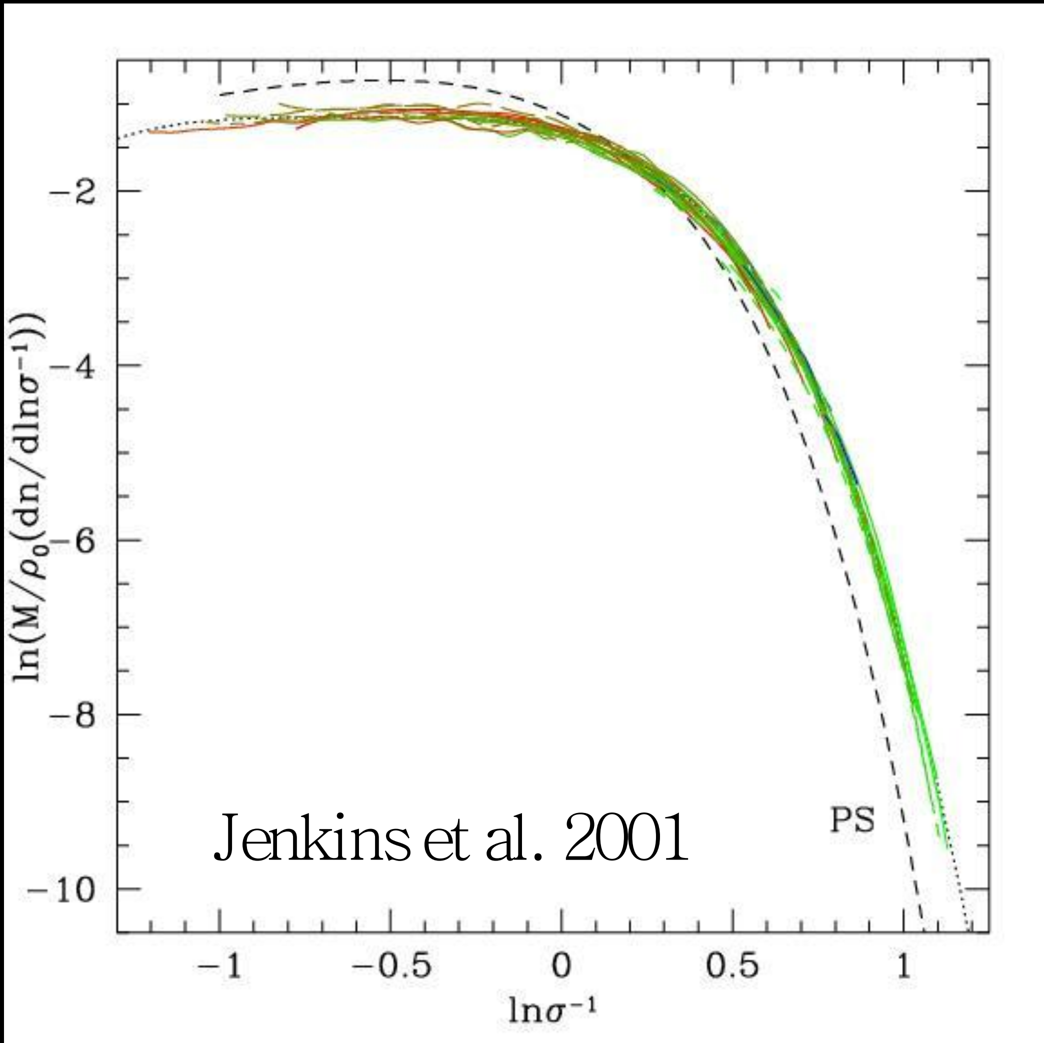
Massive halos more strongly clustered

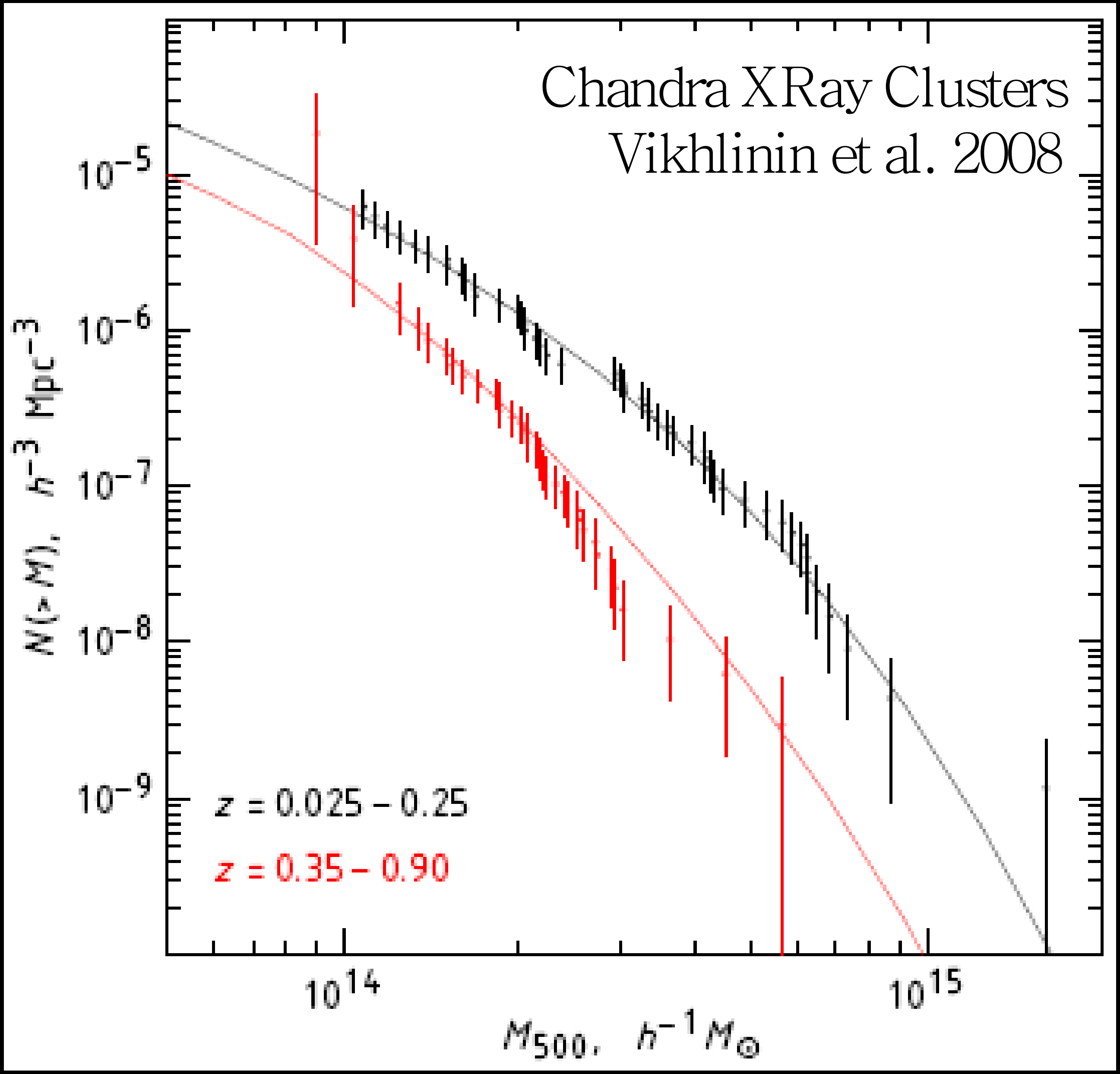
'linear' bias factor on large scales increases monotonically with halo mass



Universal form?

- Spherical evolution (Press & Schechter 1974; Bond et al. 1991)
- Ellipsoidal evolution (Sheth & Tormen 1999; Sheth, Mo & Tormen 2001)
- Simplifies analysis of cluster abundances (e.g. ACT)





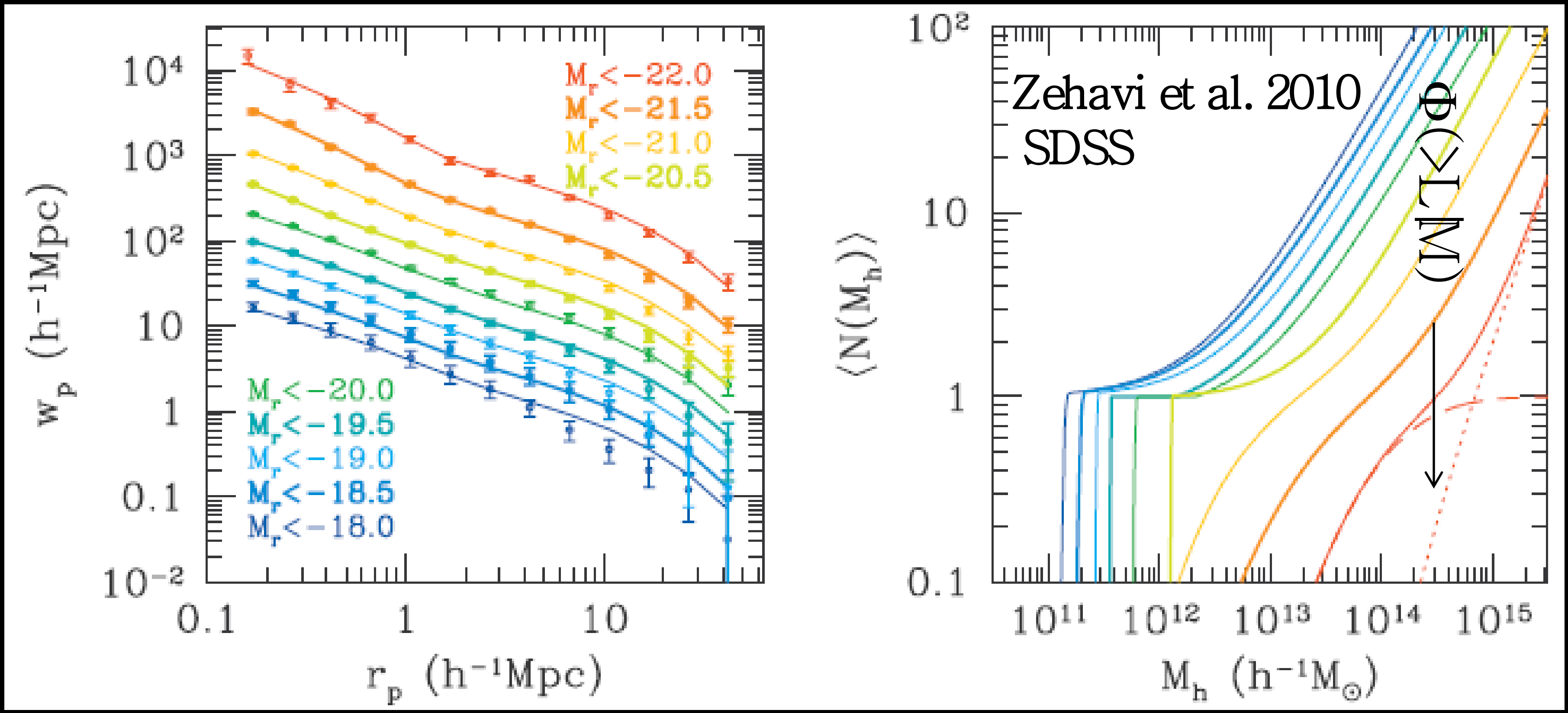
Chandra XRay Clusters
Vikhlinin et al. 2008
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
$N(>M)$, h^{-3} Mpc^{-3}
$z = 0.025 - 0.25$
$z = 0.35 - 0.90$
10^{14}
10^{15}
M_{500}, $h^{-1} M_{\odot}$

But $\delta_c \rightarrow a\,\delta_c$ with a<1

- Halo definition in simulations?
- Appropriate ensemble average?
- Fuzzy/stochastic barrier?

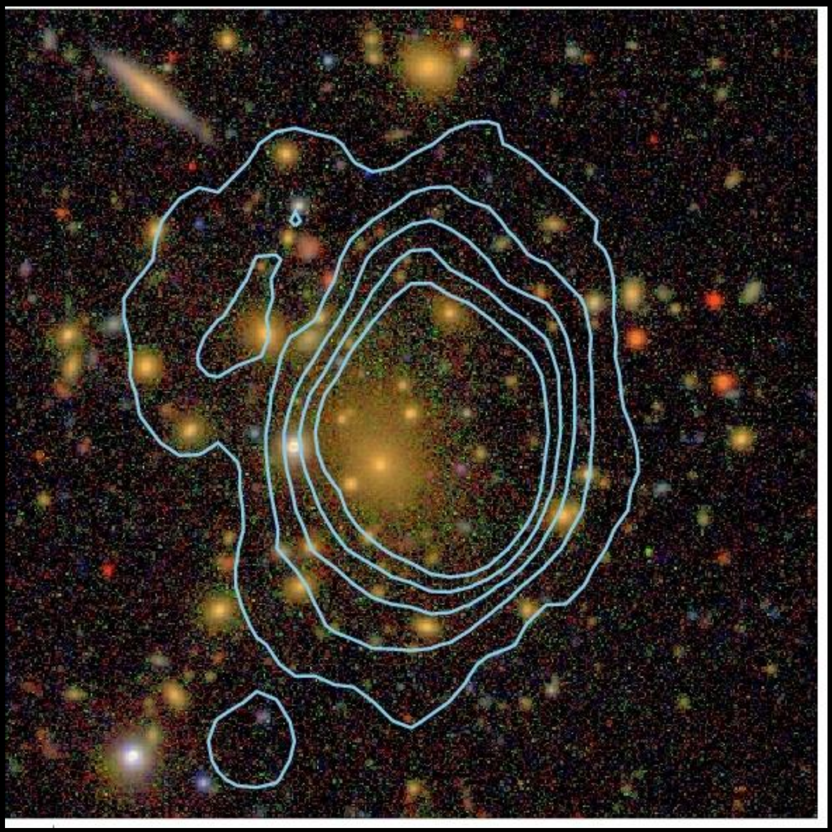
- Dependence on cosmology?

Luminosity dependence of clustering

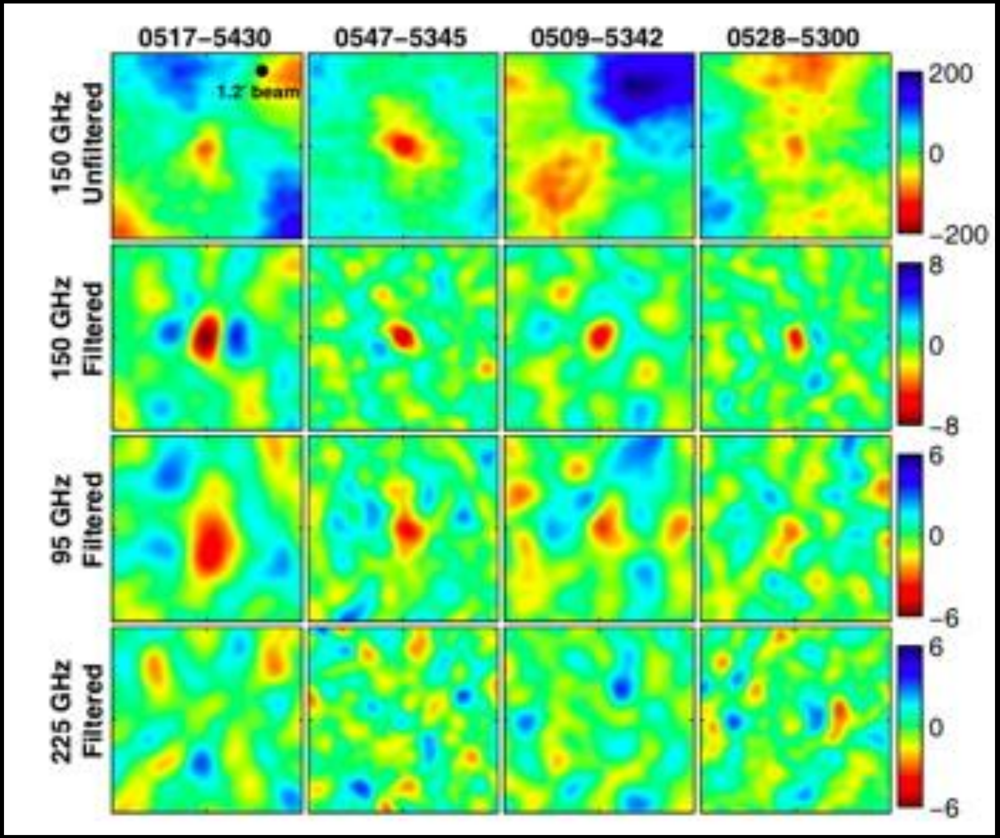


$$\langle N_{gal}|m\rangle = f_{cen}(m)\,[1 + \langle N_{sat}|m\rangle]$$

Different wavelengths return differently weighted clusters



XMM + optical



SPT

What is the weight that must be applied to each halo so that the halo catalog best represents the underlying dark matter field?

Options

- Weight each halo equally (~standard)
- Weight each halo by its bias factor
  - correct if halos are Poisson sampling of mass, a standard (and incorrect!) assumption
- Weight each halo by its mass
  - after all, we want the mass (rarely done!)
- Optimal weight must also account for missing mass (mass in 'dust')

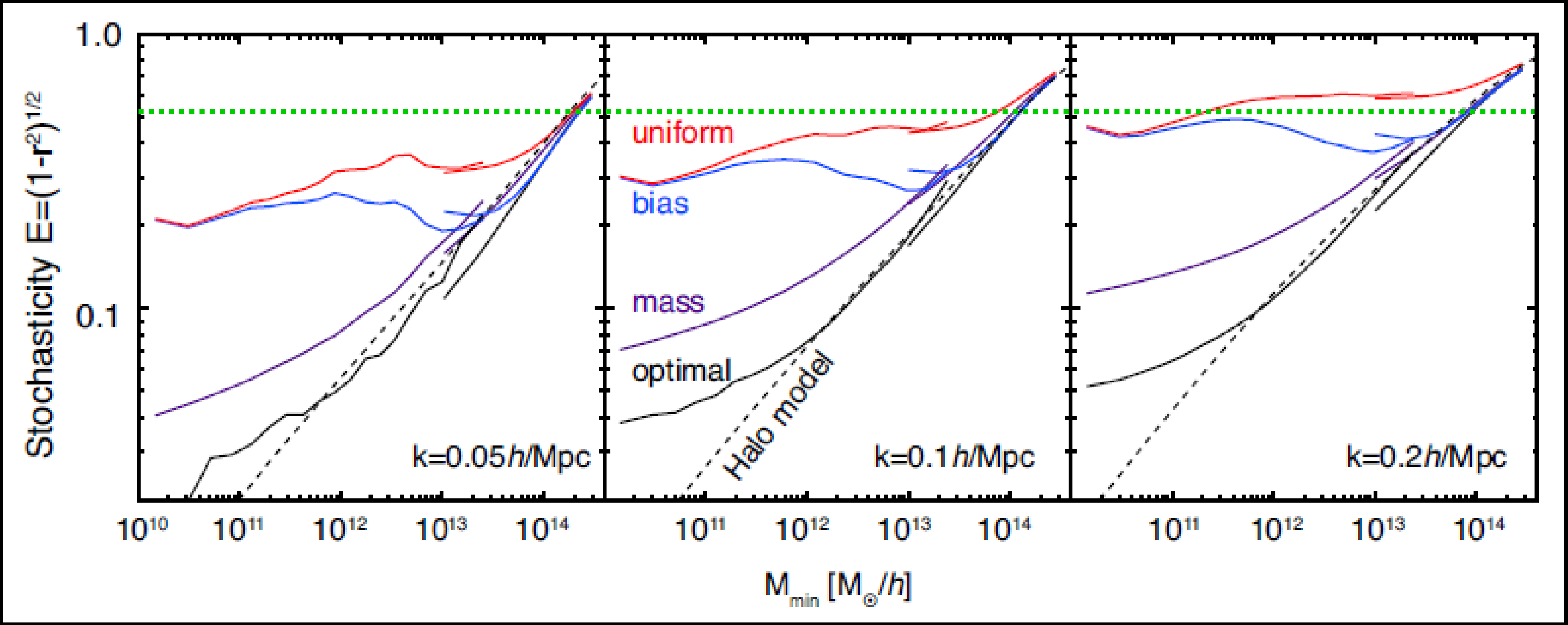
- Minimize $\sigma_w{}^2 = \langle(w\,h - bm)^2\rangle$ (Hamaus et al. 2010)
- Minimize $E^2 = \langle(m - w\,h)^2\rangle/\langle m^2\rangle$ (Cai et al. 2010)

Mass is mass-weighted halos

- Write 'Wiener filter' of model in which some halos are seen, others are not
- Stochasticity $E^2 = 1 - C_{wm}{}^2/C_{ww}/C_{mm}$
- Wiener 'filter' is that weight which minimizes stochasticity:

$$w(m) = m/\rho + b_{dust}\, b(m)\, P_h/[1 + \sum nb^2\, P_h]$$

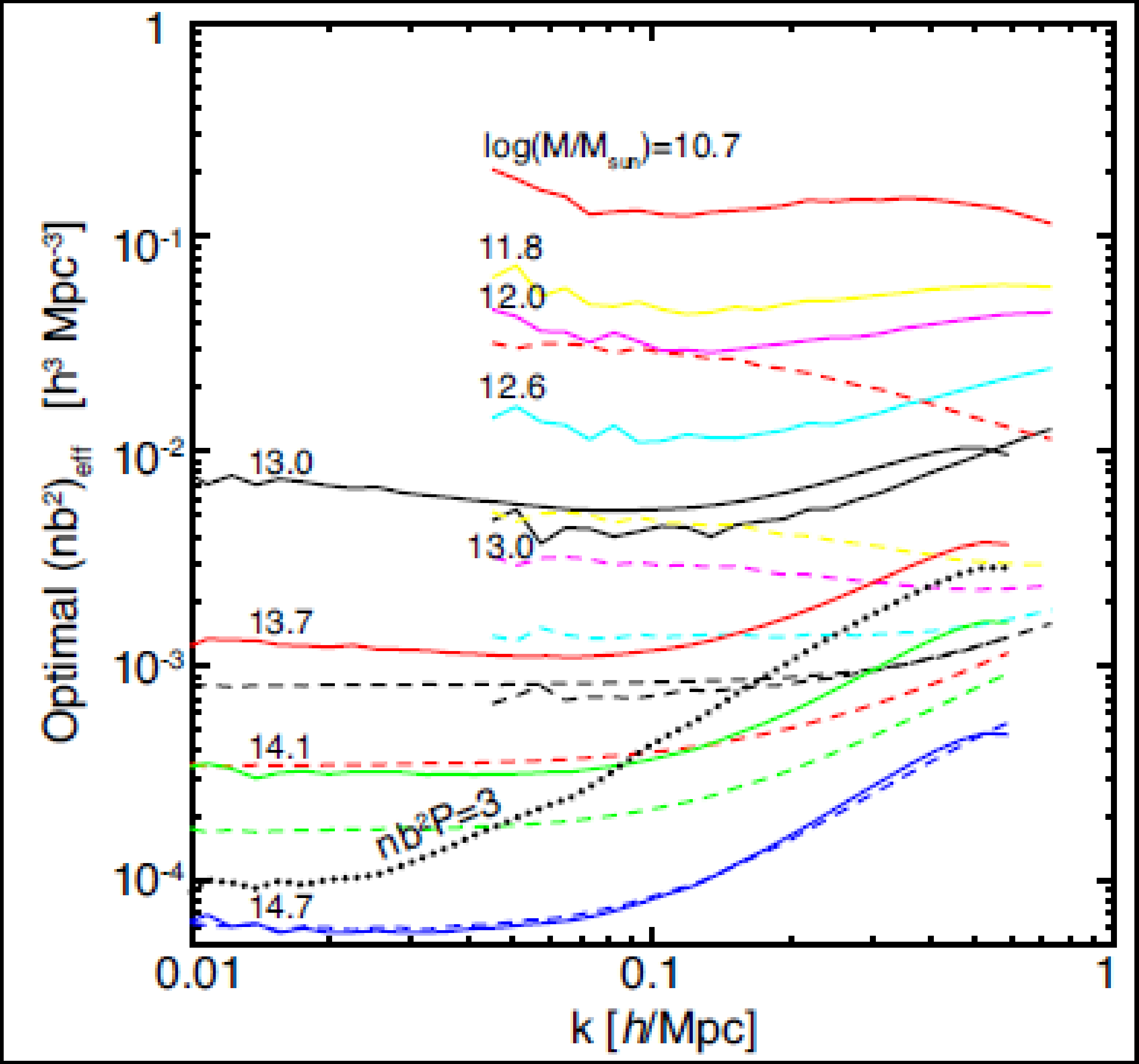
Considerable gains at low masses



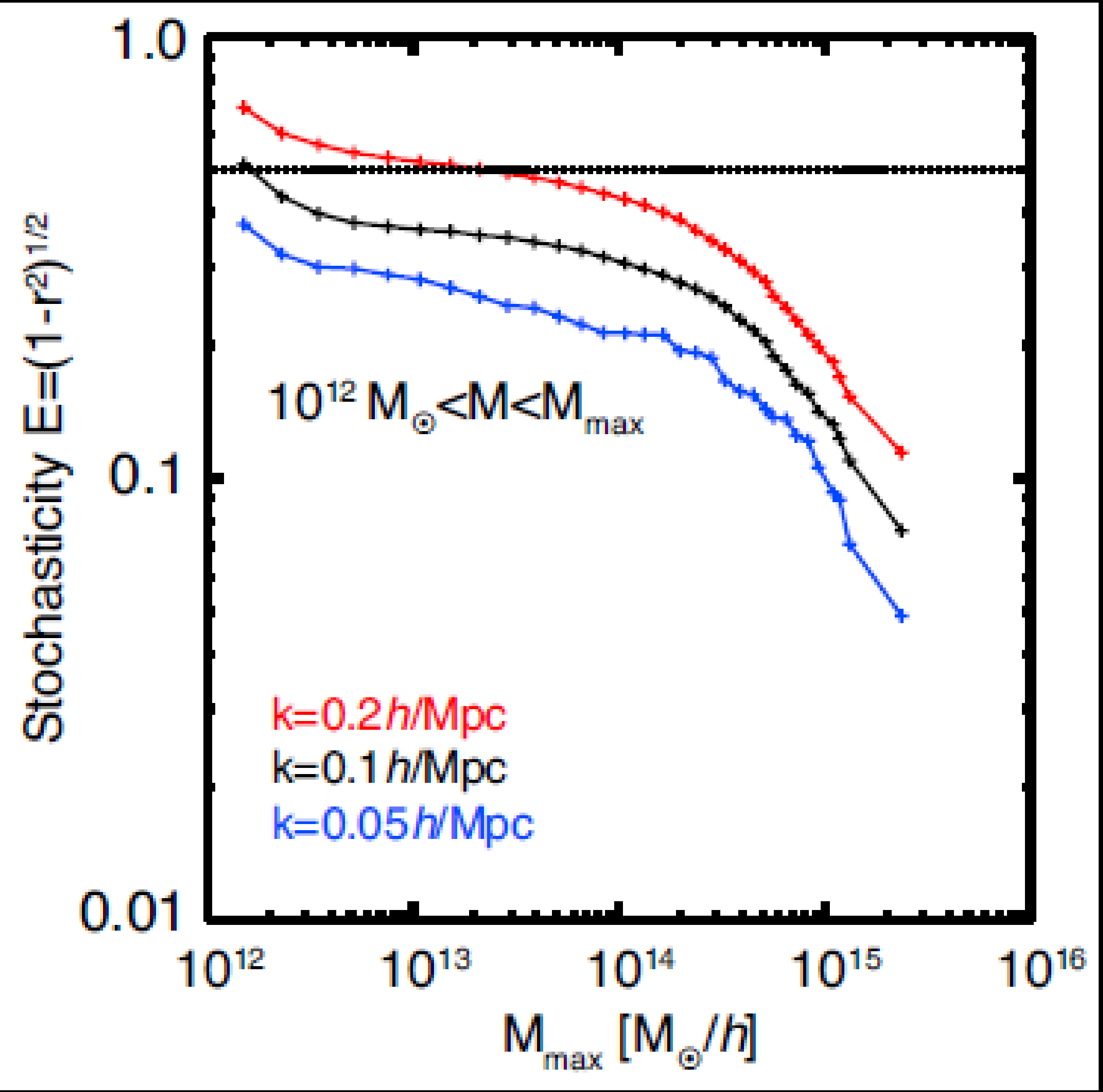
$$E^2 = N/(S + N) = 1/(S/N + 1) = 1/(nb^2 P + 1)$$

Optimal weighting yields same precision with fewer objects

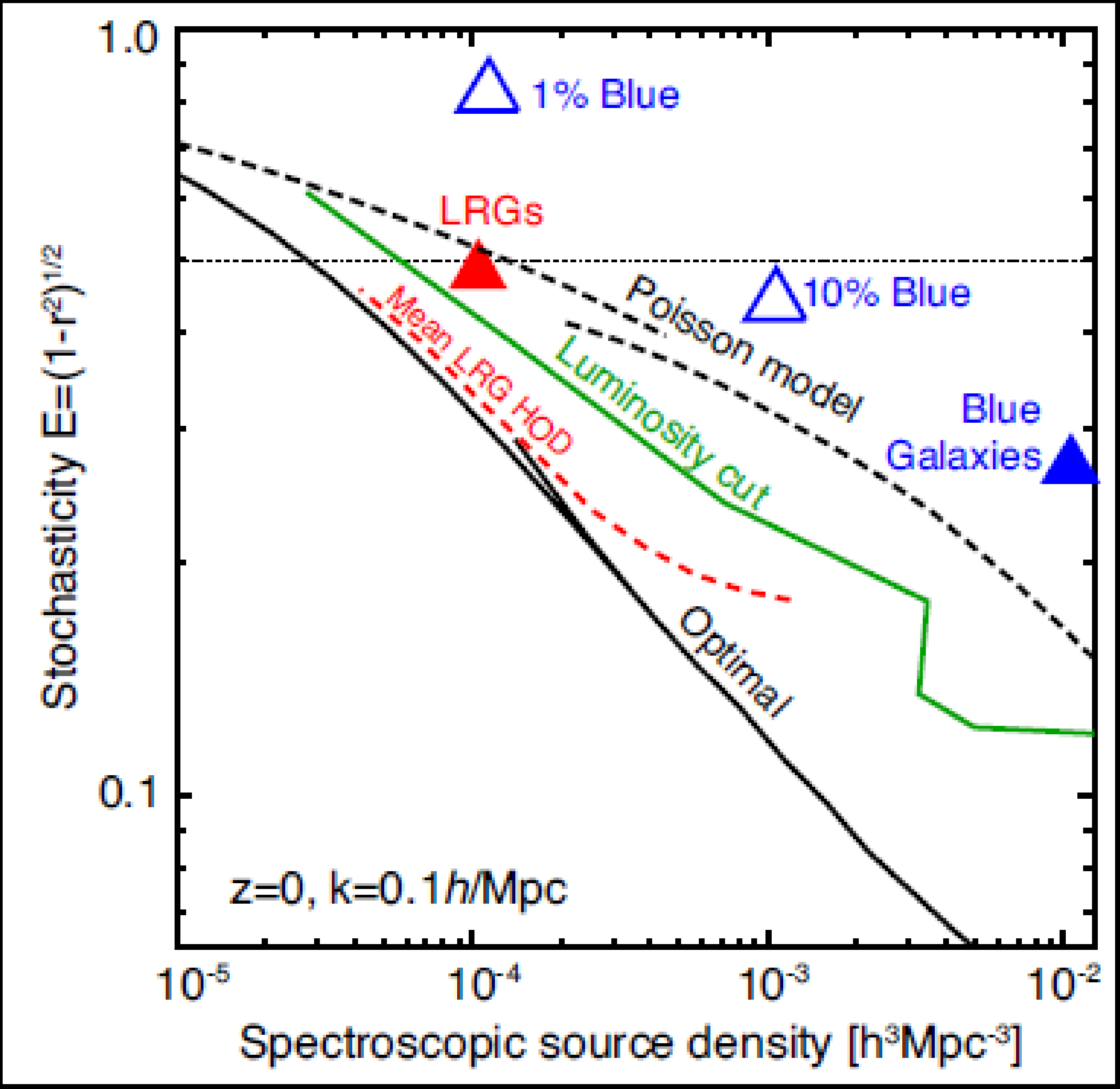
$(nb^2)_{eff}\,P$
$= 1/E^2 - 1$
$= 3$
gives
'volume
limited'
estimate of
power
spectrum



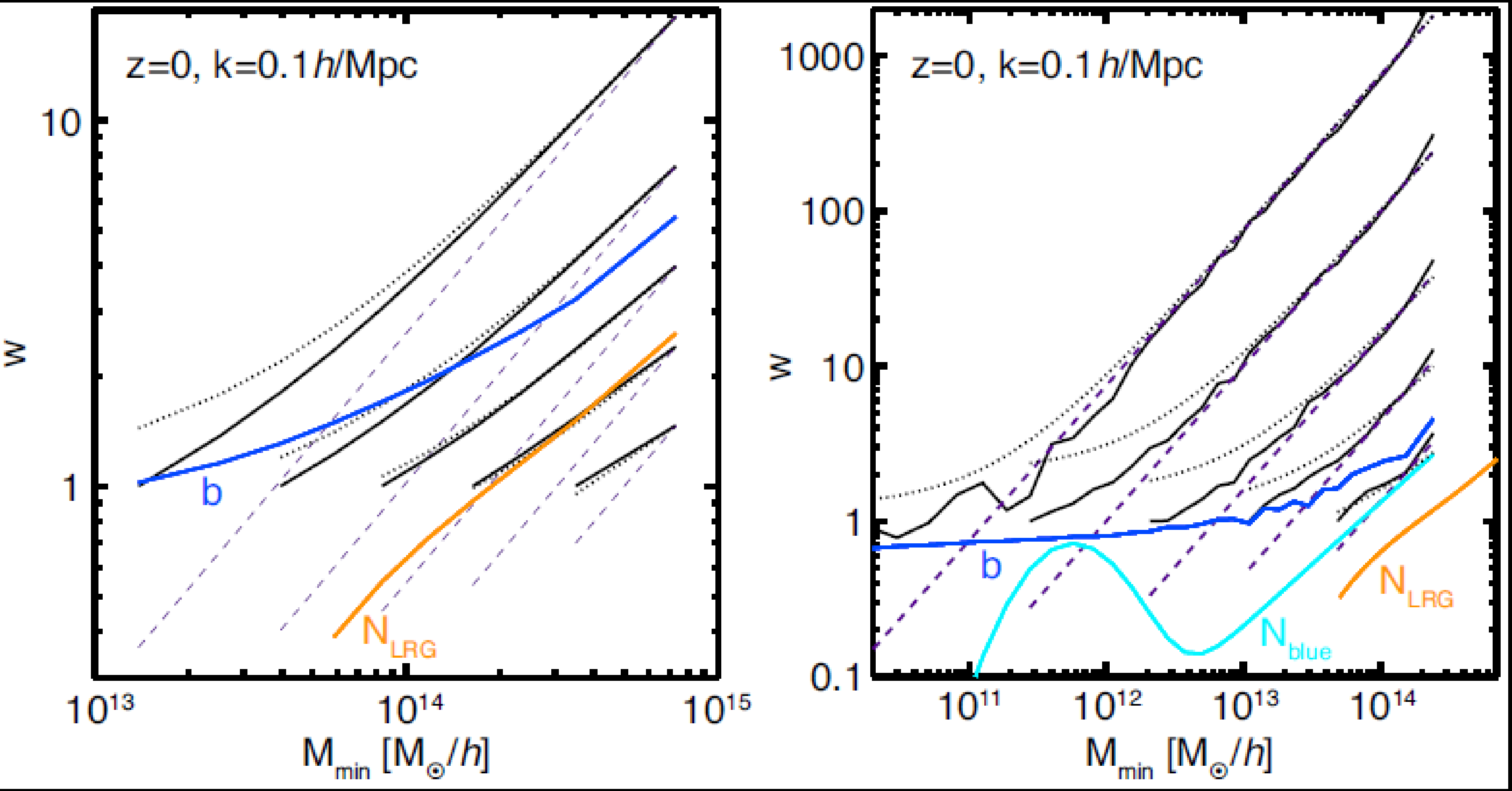
Not targeting massive halos is a bad idea



Targeting galaxies which prefer low mass halos is inefficient (costly)



Luminosity (or stellar mass) thresholded samples are not far from optimal



On going

- Easy to incorporate
  - Mass-dependent selection function
  - Uncertainty in mass estimate (N.B. this affects both m and b in optimal w)
- Determine optimal observable to use as weight (e.g., color? stellar mass?) for a given galaxy sample
- Redshift space effects/reconstructions
  - N.B. $\sigma_b/b = (E/\sqrt{2})\ (\sigma_P/P)$
- Effect of nonlinear bias, weight functions

There is much to be gained by thinking of different galaxy types and properties as simply representing the effect of applying different weights to the same underlying halo catalog

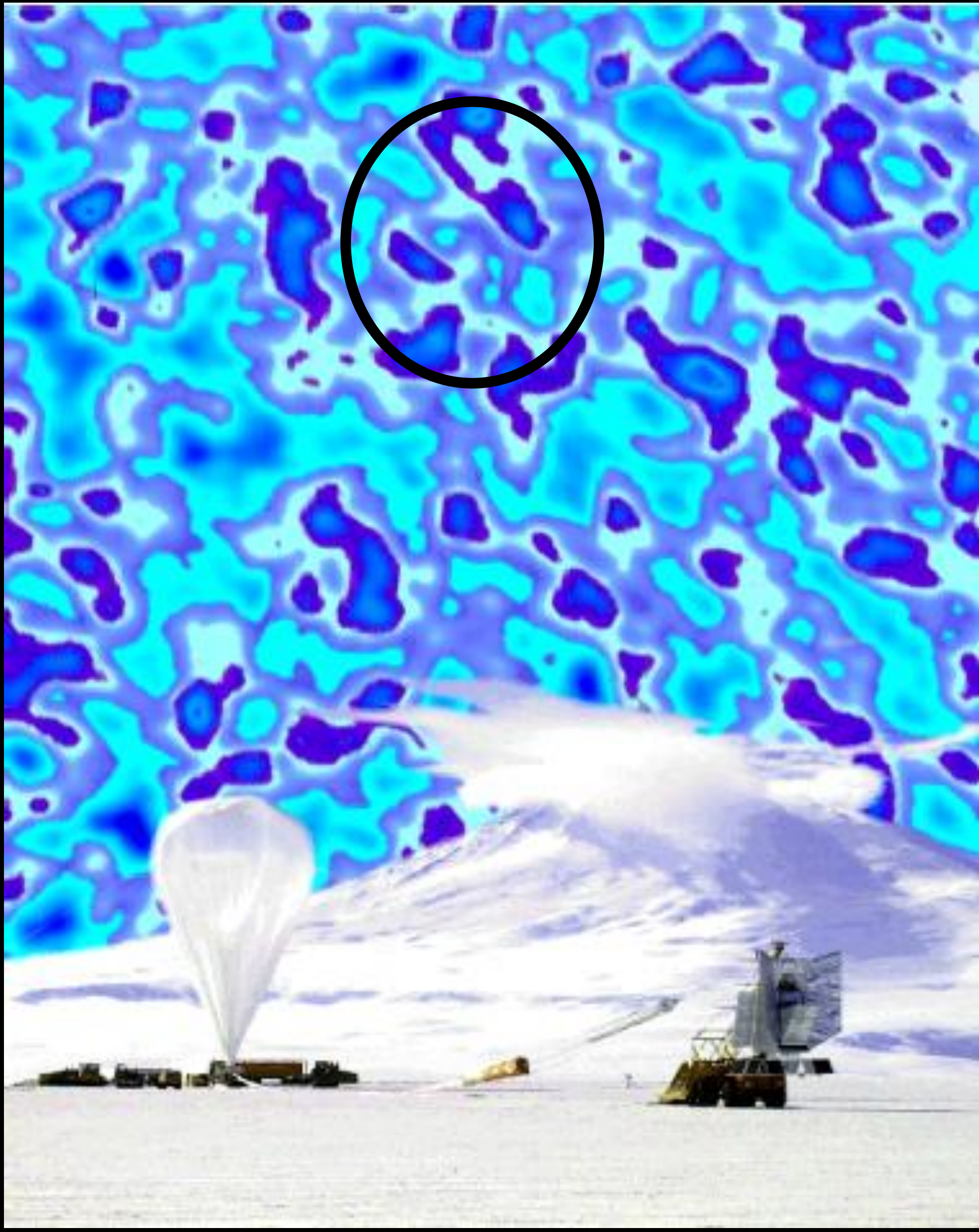
THE EXCURSION SET APPROACH

Halo abundances: Epstein (1983); Bond et al. (1991)
Hale mergers/formation: Lacey & Cole (1993)
Clustering/environment: Mo & White (1996)
Counts-in-cells: Sheth (1998); Lam & Sheth (2008)
Voids: Sheth & van de Weygaert (2004); Paranjape et al. (2011)
Filaments and sheets: Shen et al. (2006)

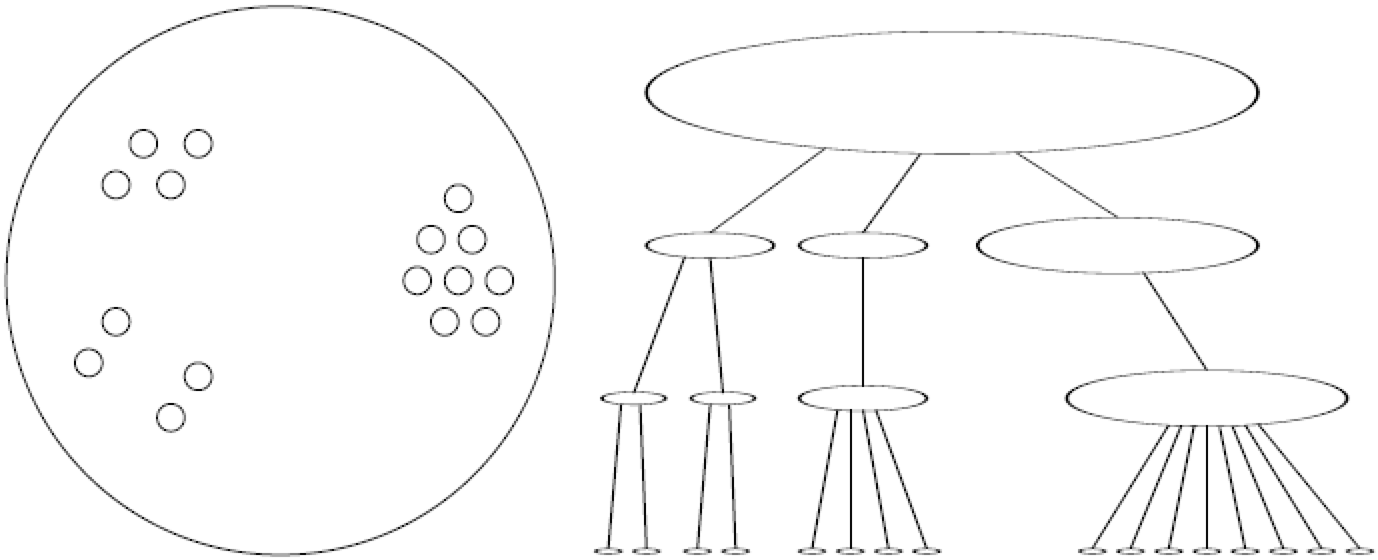
The Cosmic Background Radiation

Cold: 2.725 K
Smooth: 10^{-5}
Simple physics

Gaussian fluctuations
= seeds of subsequent structure formation
= simple(r) math
Logic which follows is general



Initial conditions determine merger history



(Mo & White 1996; Sheth 1996)

Birkhoff's theorem important

Use initial conditions (CMB)

+

model of nonlinear gravitational clustering to make inferences about late-time, nonlinear structures

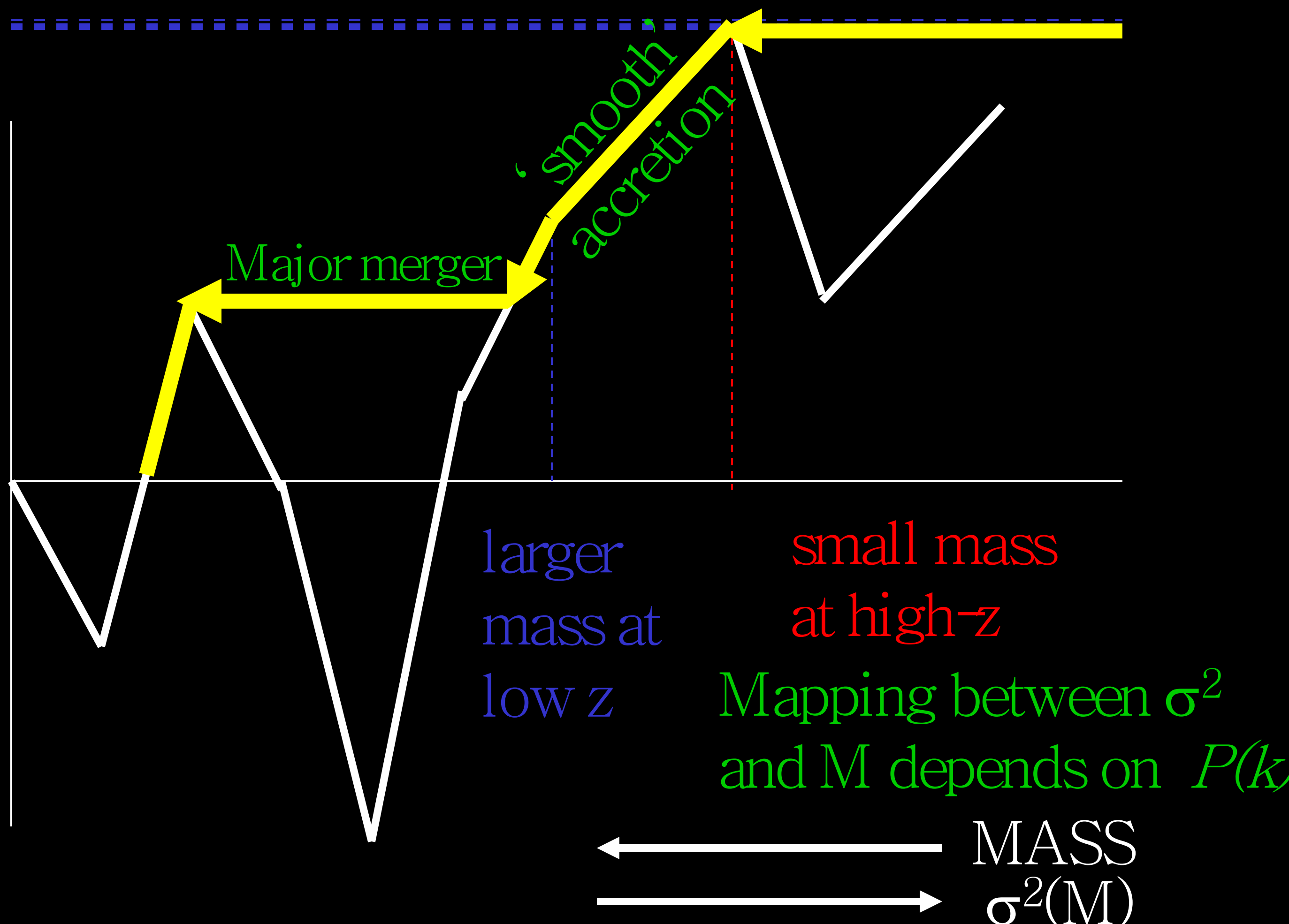
The excursion set approach

High-z

Low-z

over-density

Time evolution of barrier depends on cosmology



Spherical evolution very well approximated by 'deterministic' mapping ⋯

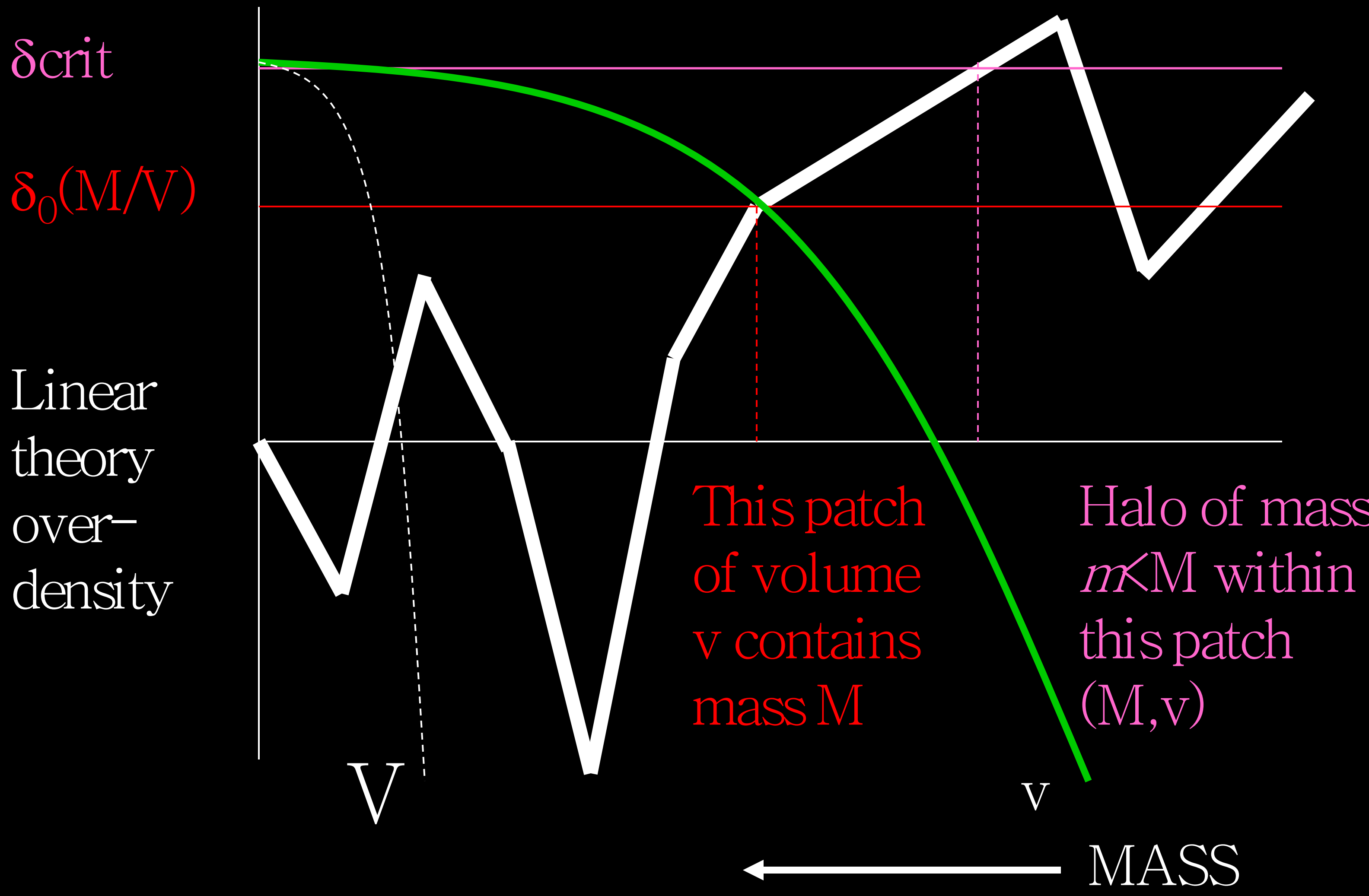
$$(R_{\rm initial}/R)^3 = \text{Mass}/(\rho_{\rm com}\text{Volume}) =$$

$$1+\delta \approx (1-\delta_0/\delta_{sc})^{-\delta sc}$$

⋯which can be inverted:

$$(\delta_0/\delta_{sc}) \approx 1-(1+\delta)^{-1/\delta sc}$$

The Nonlinear PDF



- Halo mass function is distribution of counts in cells of size $v \to 0$ that are not empty.

- Fraction f of walks which first cross barrier associated with cell size V at mass scale M,

$$f(M|V)\, dM = (M/V)\, p(M|V)\, dM$$

where $p(M|V)\, dM$ is probability randomly placed cell V contains mass M.

- Note: all other crossings irrelevant → stochasticity in mapping between initial and final density

From Walks to Halos: Ansätze

- $f(\delta_c, s)ds$ = fraction of walks which first cross $\delta_c(z)$ at s

  $\approx$ fraction of initial volume in patches of comoving volume $V(s)$ which were just dense enough to collapse at z

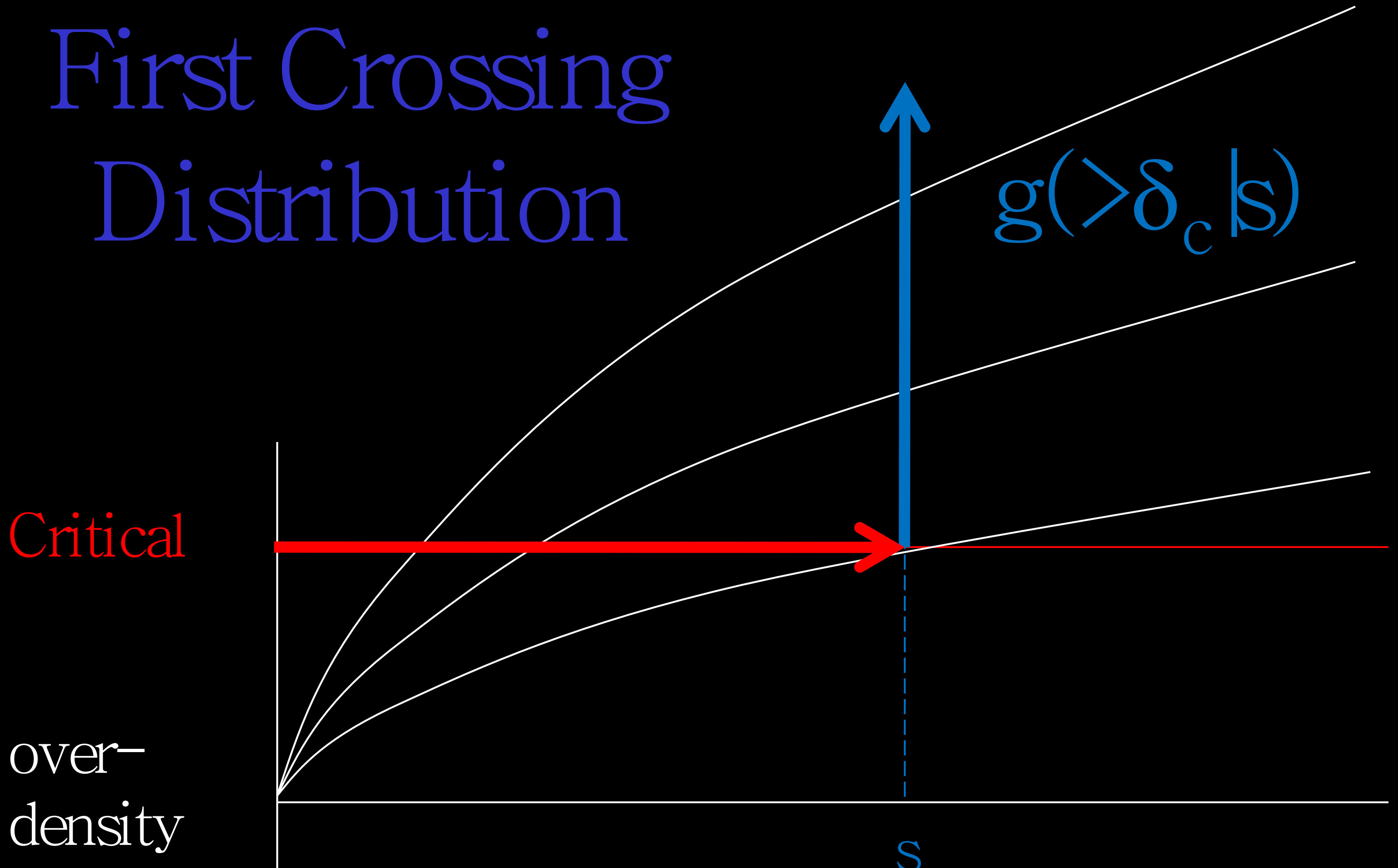
  $\approx$ fraction of initial mass in regions which each initially contained $m = \rho V(1+\delta_c) \approx \rho V(s)$ and which were just dense enough to collapse at z (ρ is comoving density of background)

  $\approx dm\, m\, n(m,\delta_c)/\rho$

Simplification because…

- Everything local
- Evolution determined by cosmology (competition between gravity and expansion)
- Statistics determined by initial fluctuation field: since Gaussian, statistics specified by initial power-spectrum *P(k)*
- Fact that only very fat cows are spherical is a detail (*crucial* for precision cosmology); in excursion set approach, mass-dependent barrier height increases with distance along walk

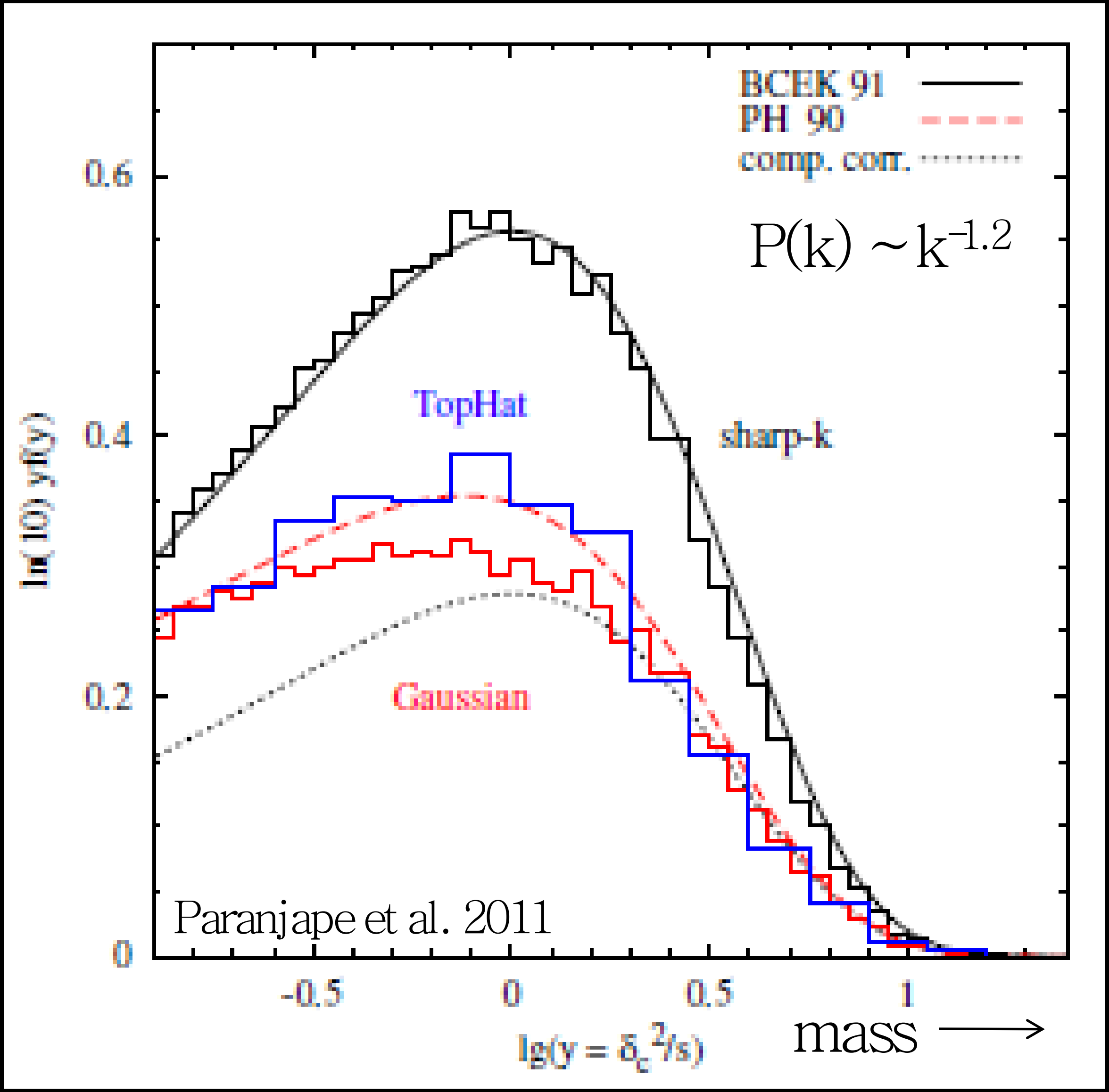
First Crossing Distribution

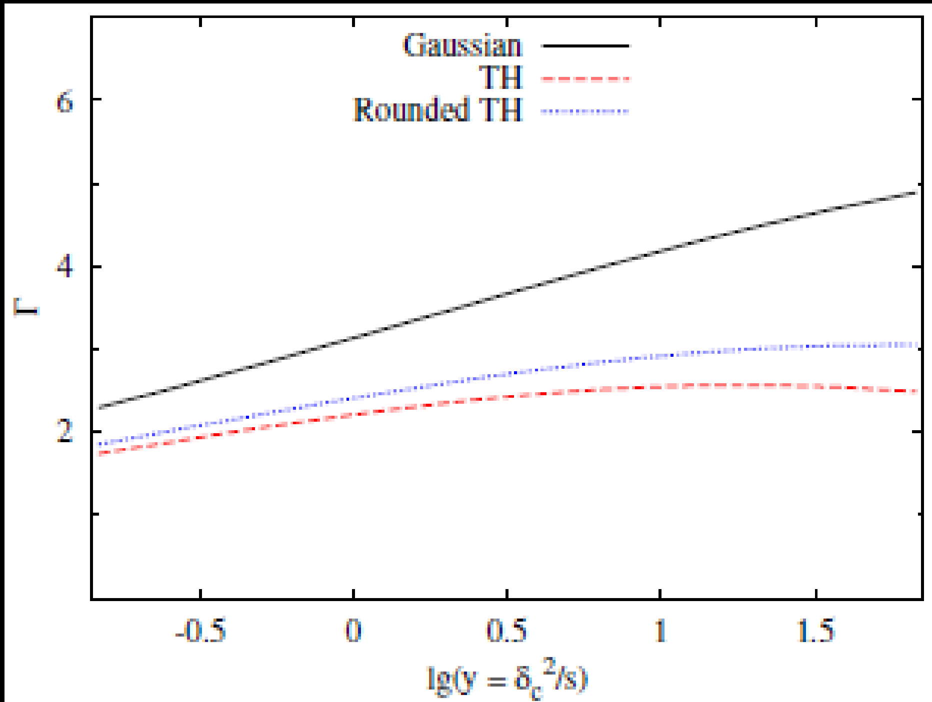


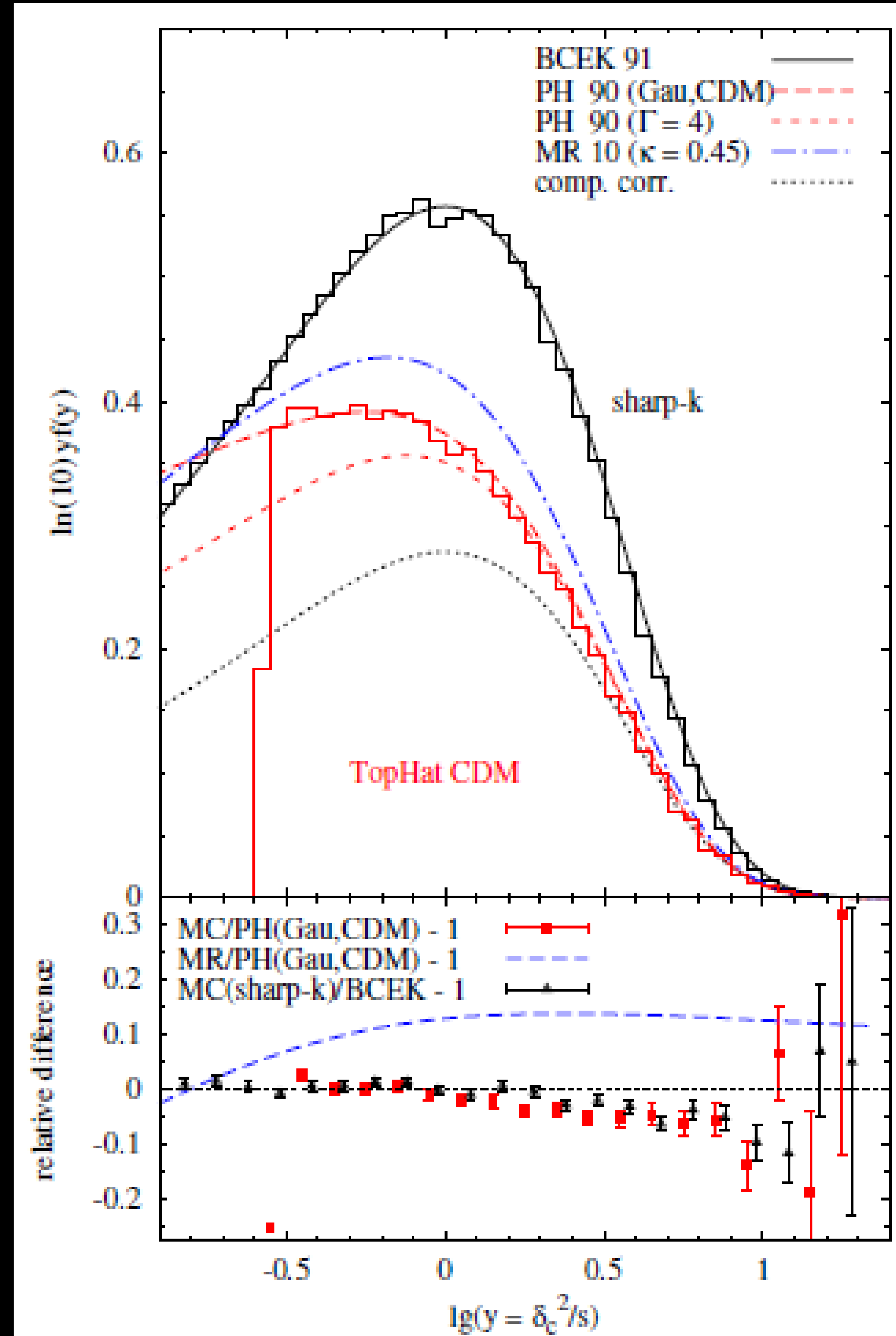
$$ {}_0\int^{s} dS\, f(S)\, p(>\delta_c, s \,|\, \text{first } \delta_c \text{ at } S) $$

$$ \approx {}_0\int^{s} dS\, f(S)\, p(>\delta_c, s \,|\, \delta_c, S) $$

Peacock-Heavens approximation to correlated steps problem is most accurate analytic approximation to date

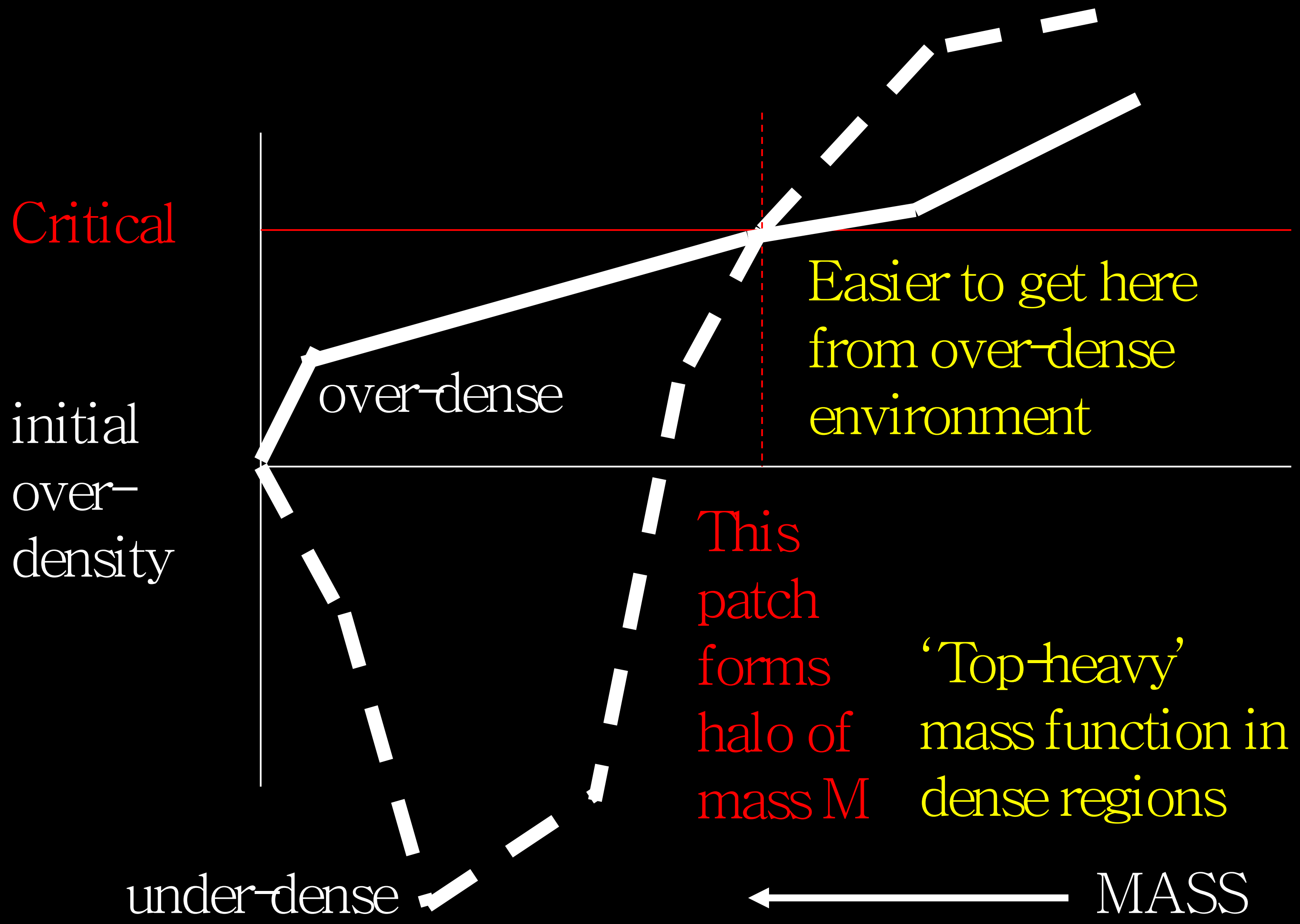






- Agreement very good whatever the shape of Pk, smoothing filter

Correlations with environment



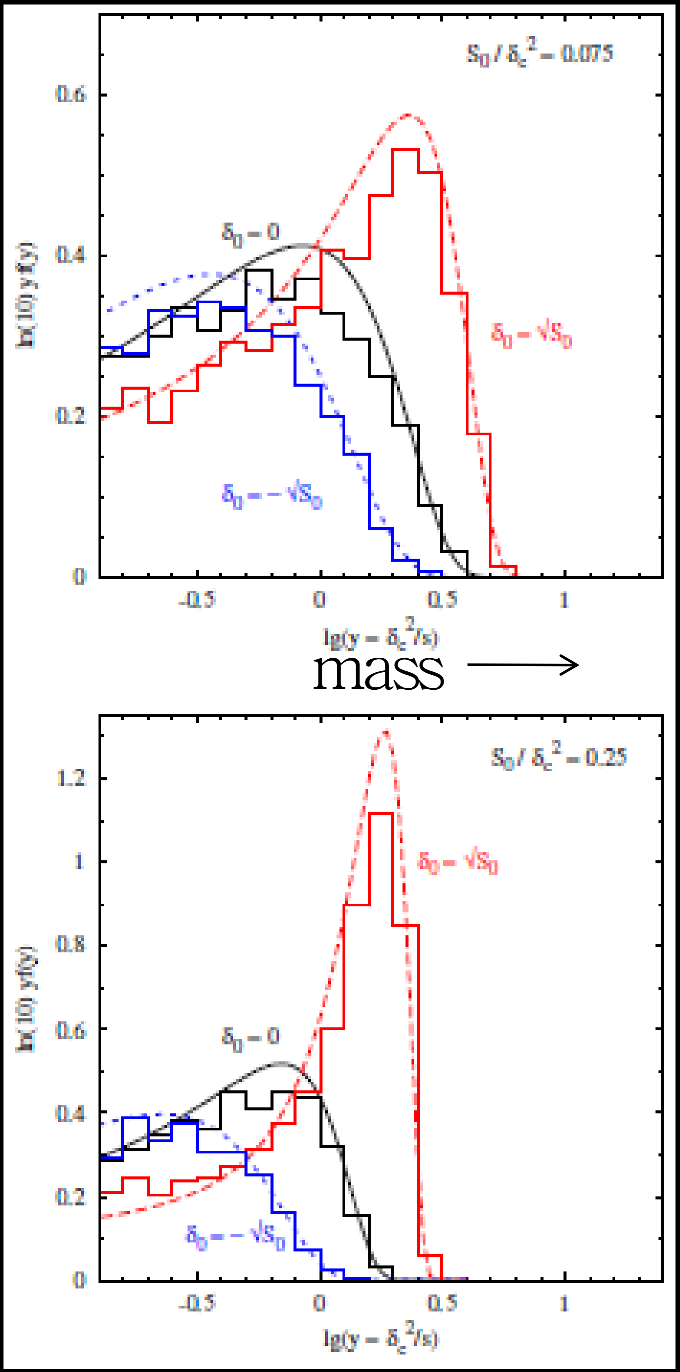
- Generalization to constrained walks:

  $p(<\delta_c|s) \rightarrow p(<\delta_c, s|\delta_0, S_0)$

  is trivial and accurate

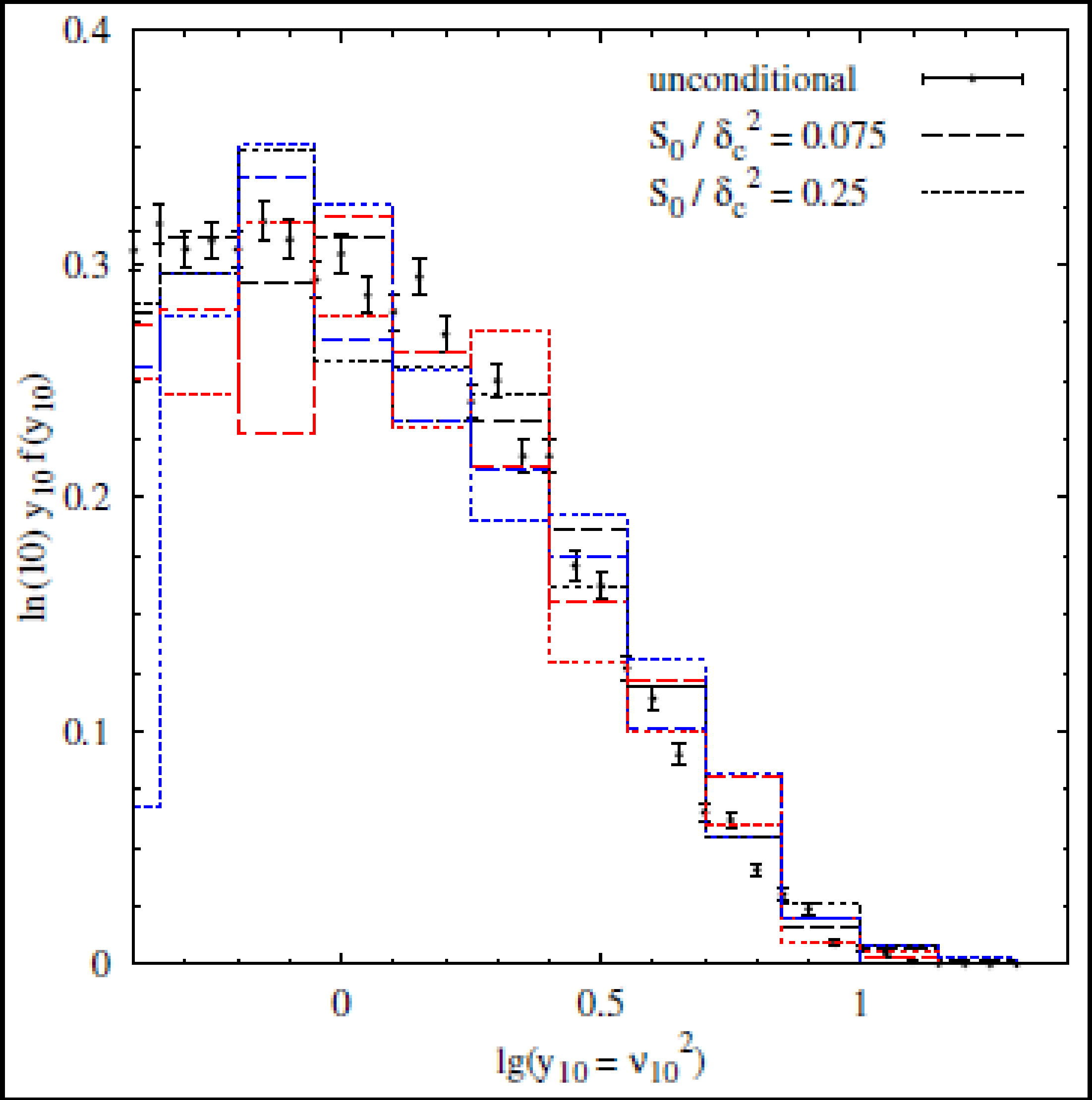
$$\nu_{10} = \frac{\delta_1 - (S_x/S_0)\,\delta_0}{\sqrt{S_1 - (S_x/S_0)^2 S_0}}$$

- Read fine print (and correct the typos!) in Bond et al. (1991)



$$\nu_{10} = \frac{\delta_1 - (S_x/S_0)\,\delta_0}{\sqrt{S_1 - (S_x/S_0)^2\, S_0}}$$

N.B.: $S_x/S_0 \neq 1$



Large scale bias

- $f(m|\delta_0)/f(m) - 1 \sim f(\nu_{10})/f(\nu) - 1$
  $\sim (S_x/S)\, b(\nu)\, \delta_0$

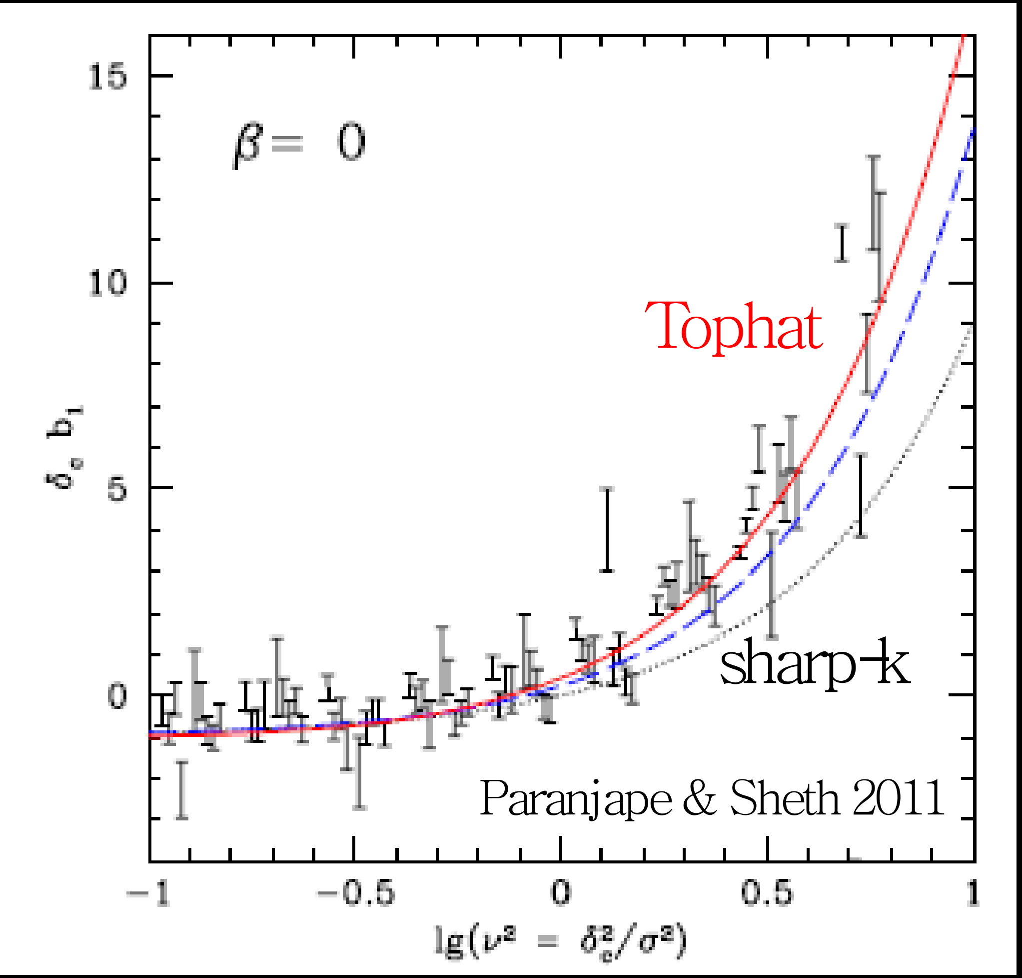
  where b is usual (Mo-White) bias factor
- So, real space bias should differ from Fourier space bias by factor (S_x/S)
- N.B. exactly same effect seen for peaks (Desjacques et al. 2010); in essence, this is same calculation as the 'peak' density profile (BBKS 1986)

Real and Fourier space measures of bias should differ, especially for most massive halos

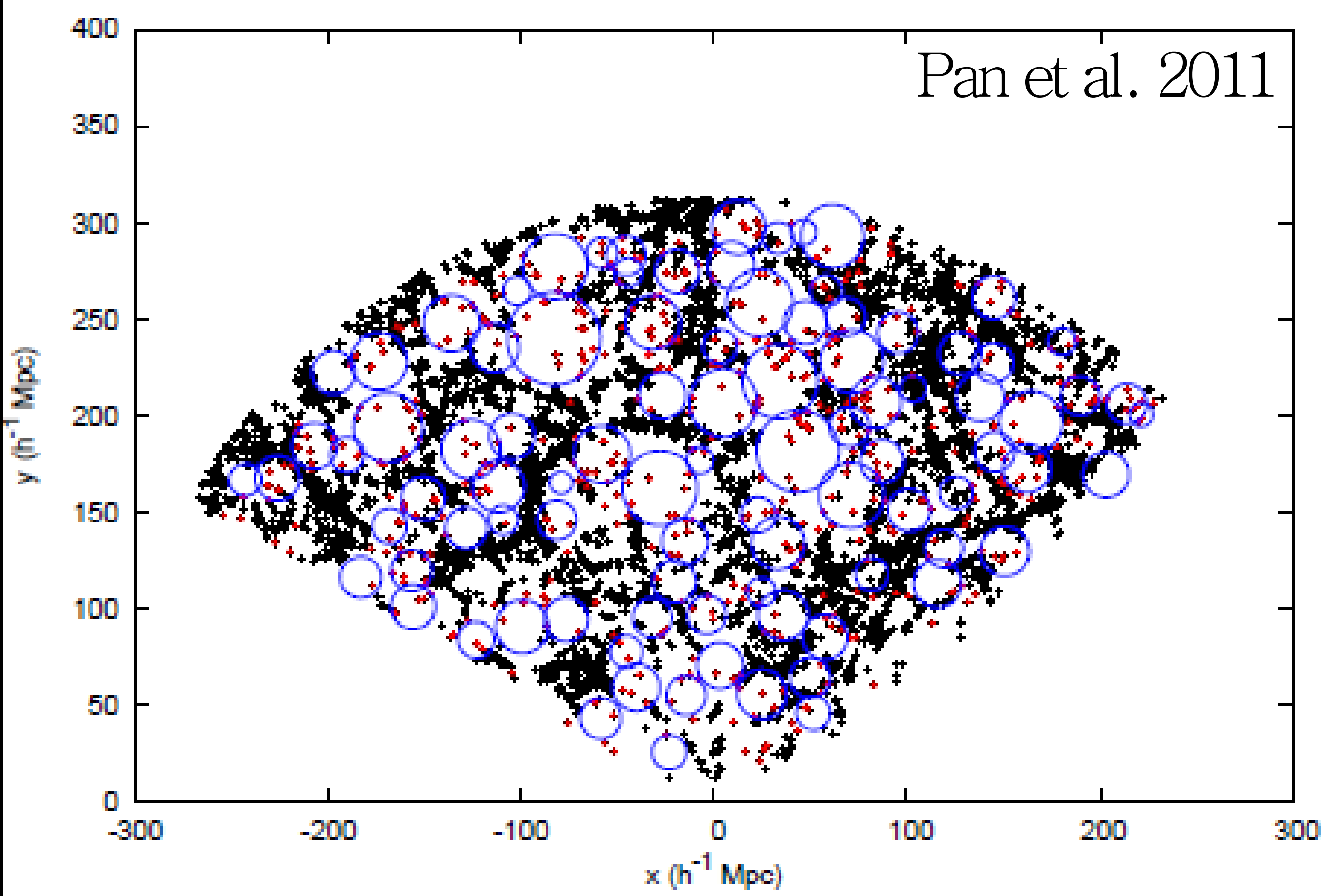
Basis for getting scale dependent bias from excursion set approach

Assembly bias from

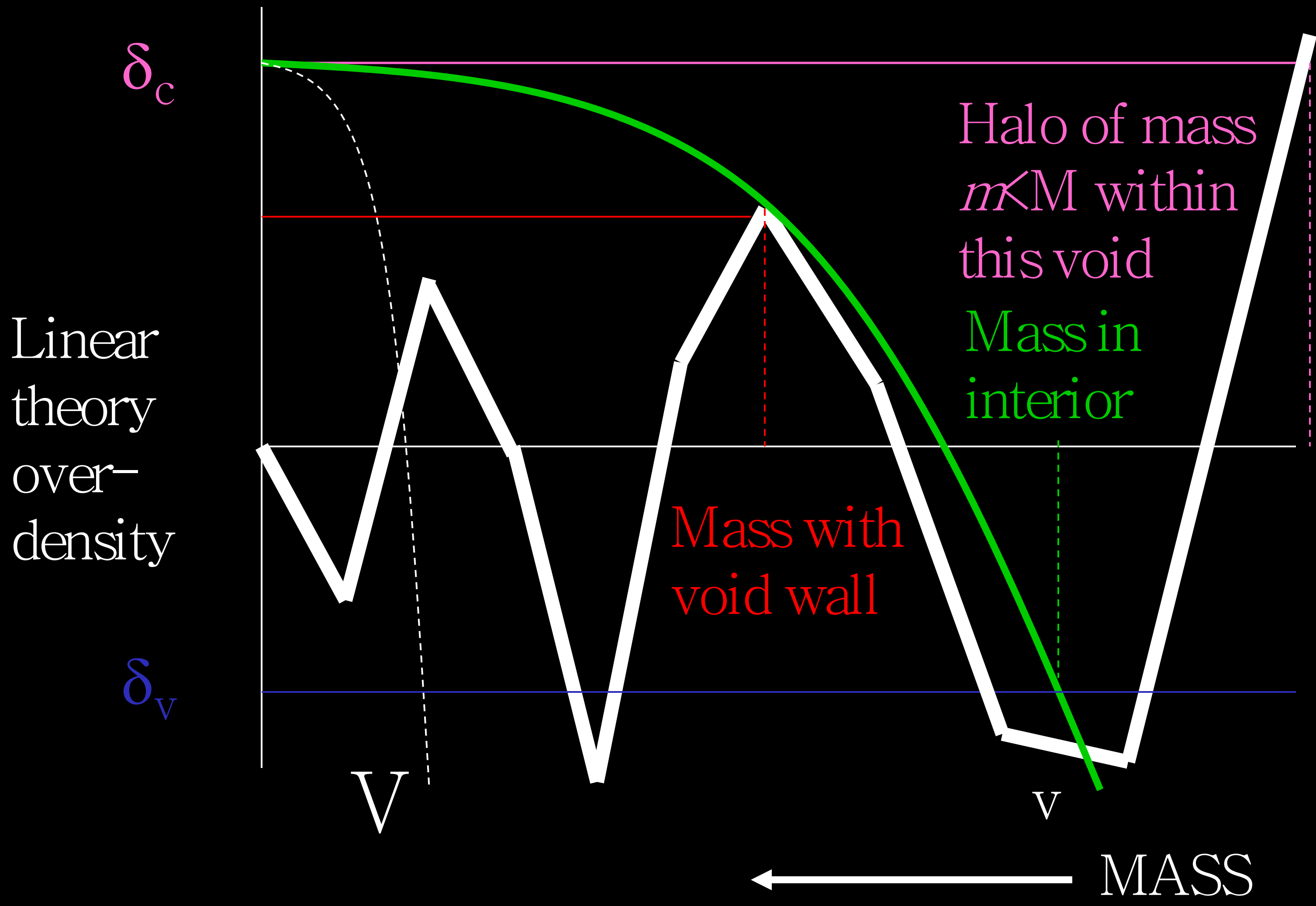
$\langle \delta_0, S_0 \,|\, \delta_c, S_1, \delta_f, S_f \rangle$



Voids in SDSS



Voids

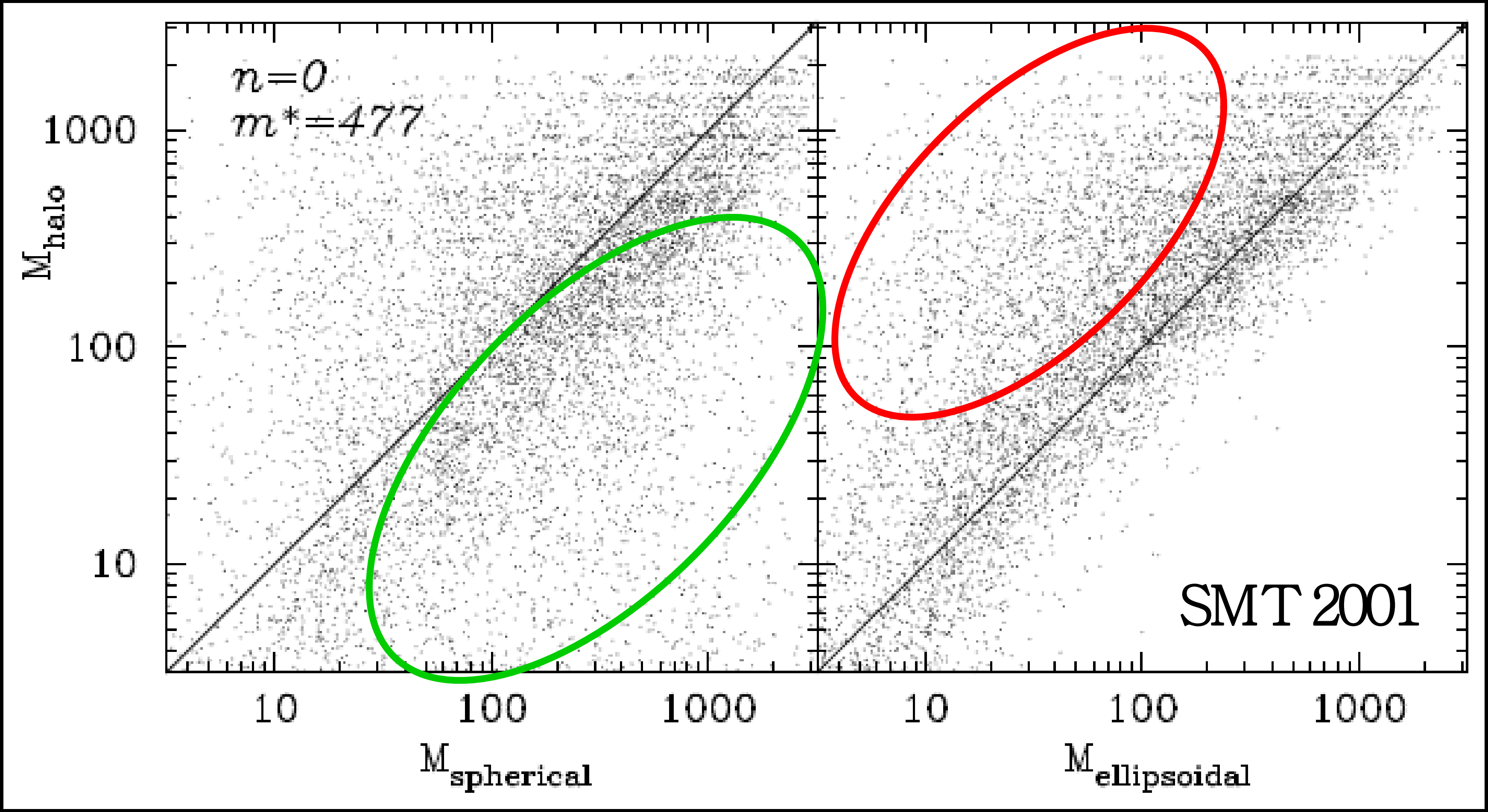


δ_c
Halo of mass
m<M within
this void
Mass in
interior
Linear
theory
over-
density
Mass with
void wall
δ_v
V
v
MASS

Darker issues

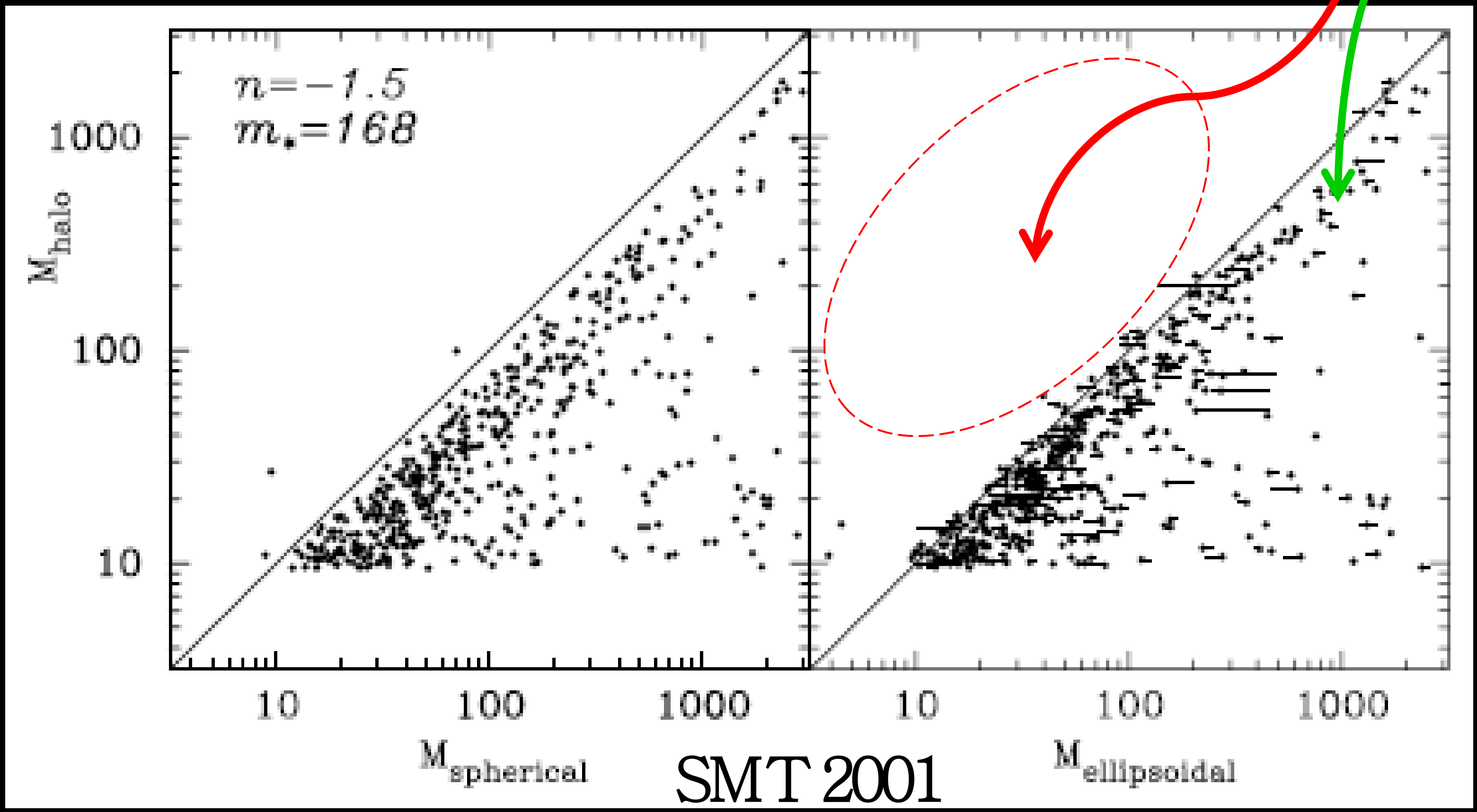
- In addition to differences between real and k-space bias, expect differences between bias factors associated with cross- and auto-correlations
- Origin of fudge ($\delta_c \rightarrow a\,\delta_c$) factor?

Spherical collapse not full story



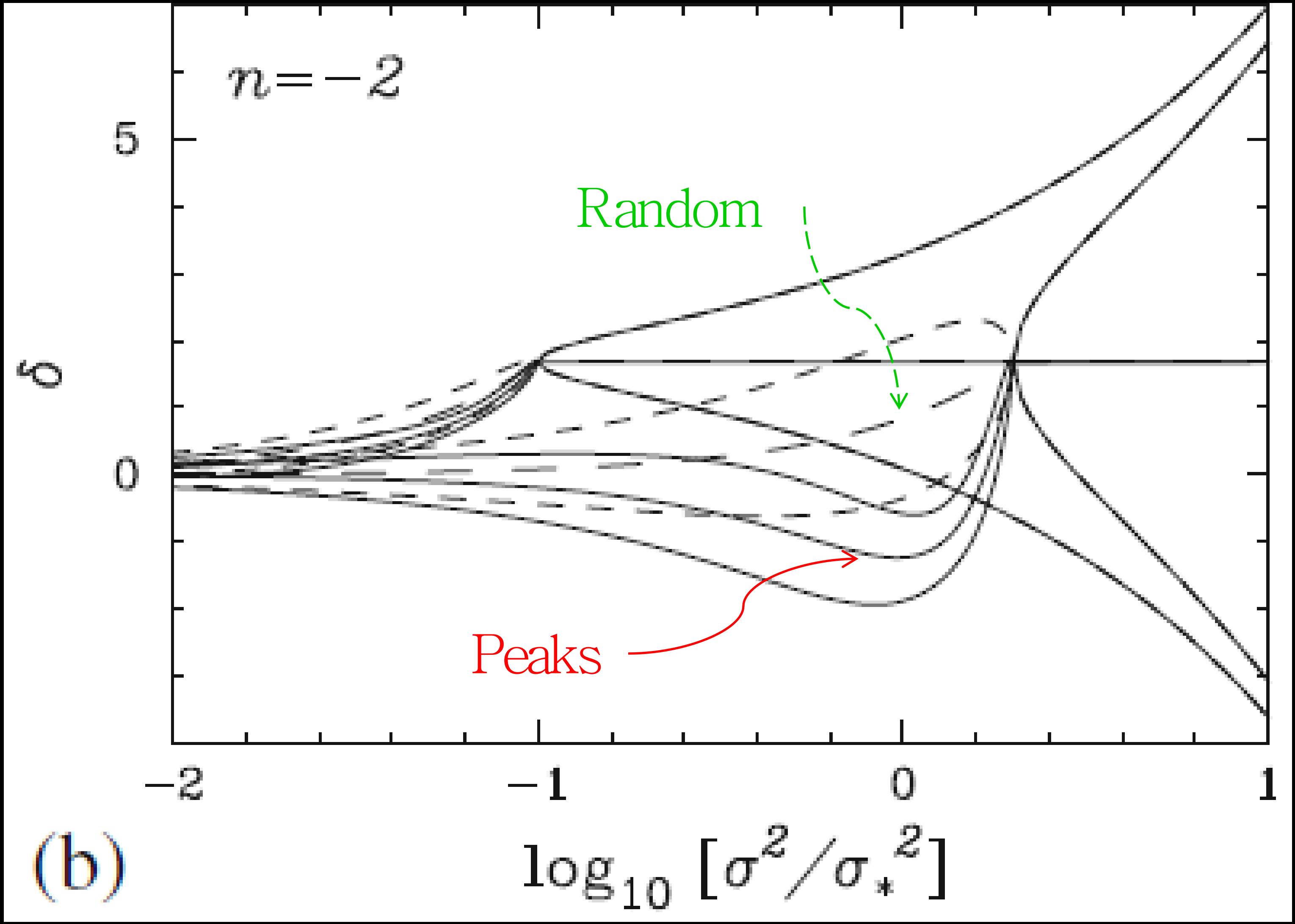
$M_{sim} > M_{predicted}$ often

Walks centered on center of mass particle are not same as walks centered on random positions



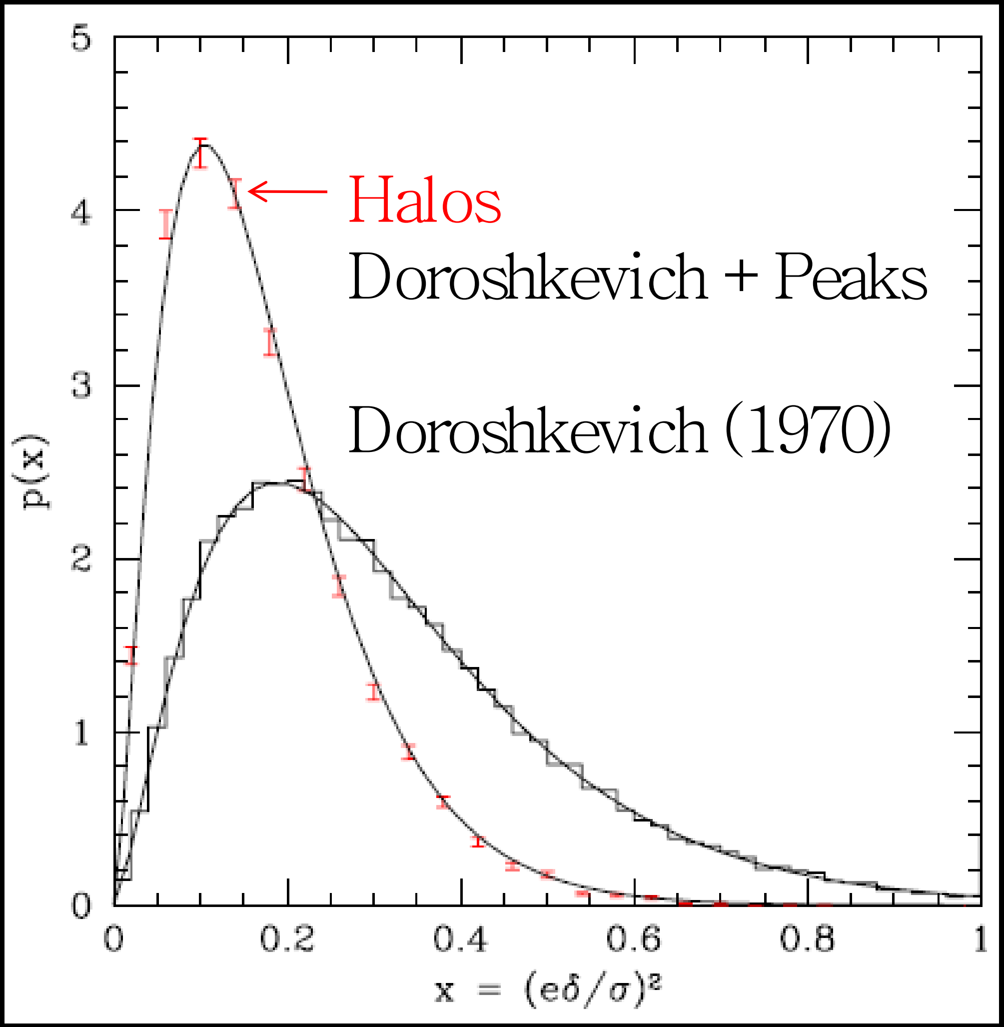
SMT 2001

The peaks model



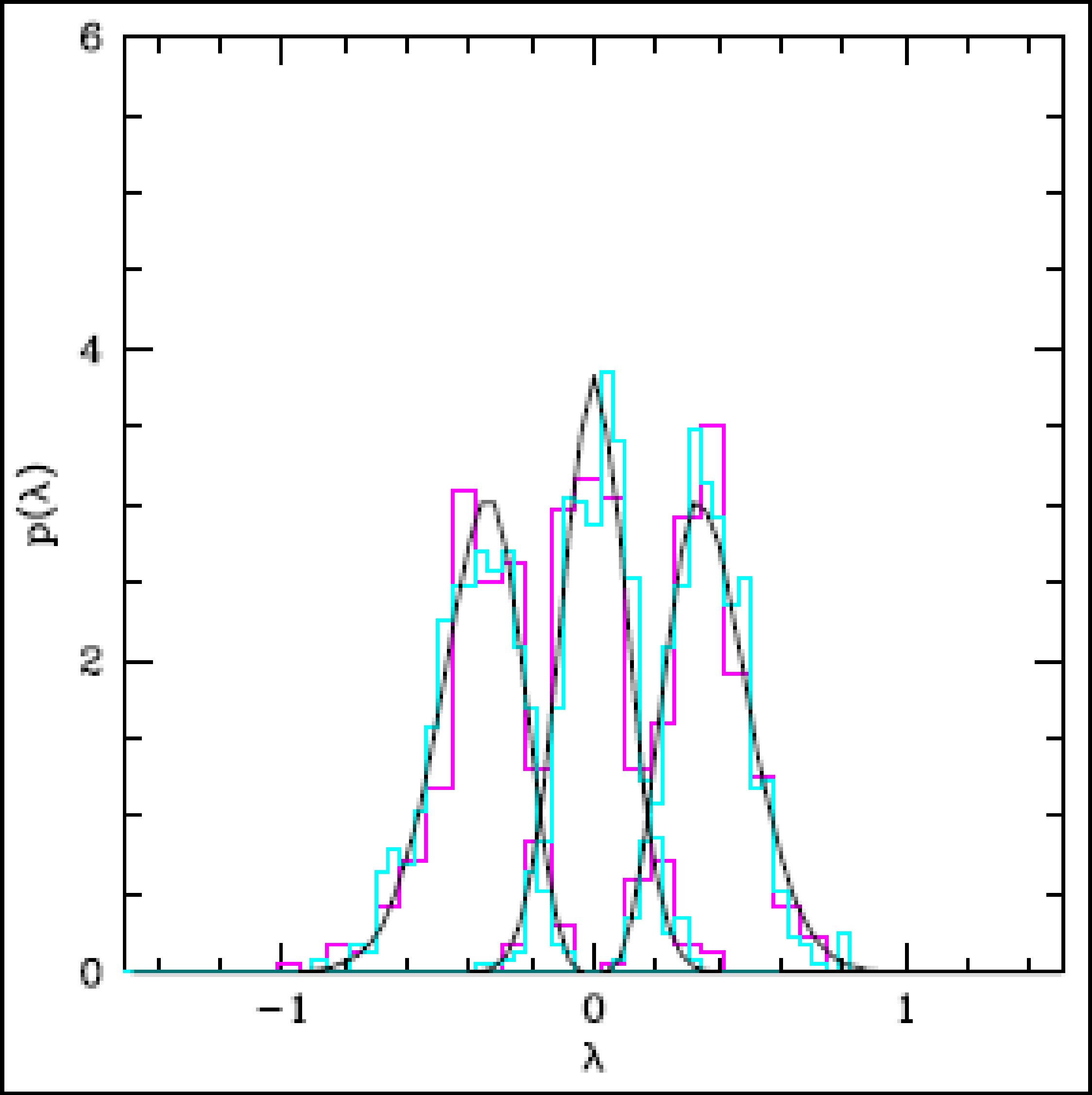
Other, more direct evidence from statistics of initial patches

- For EC, need $p(\delta,e,p) = p(d)\ p(e,p|\delta)$
- For random patches, Doroshkevich (1970) shows $p(e,p|\delta)$ same for all δ, and distribution of $(\delta e)/\sigma(m) \sim (\lambda_1-\lambda_3)/\sigma(m)$ is universal
- In simulations, $p(\delta e/\sigma)$ indeed universal, but with smaller variance ~ like distribution around peaks in δ.

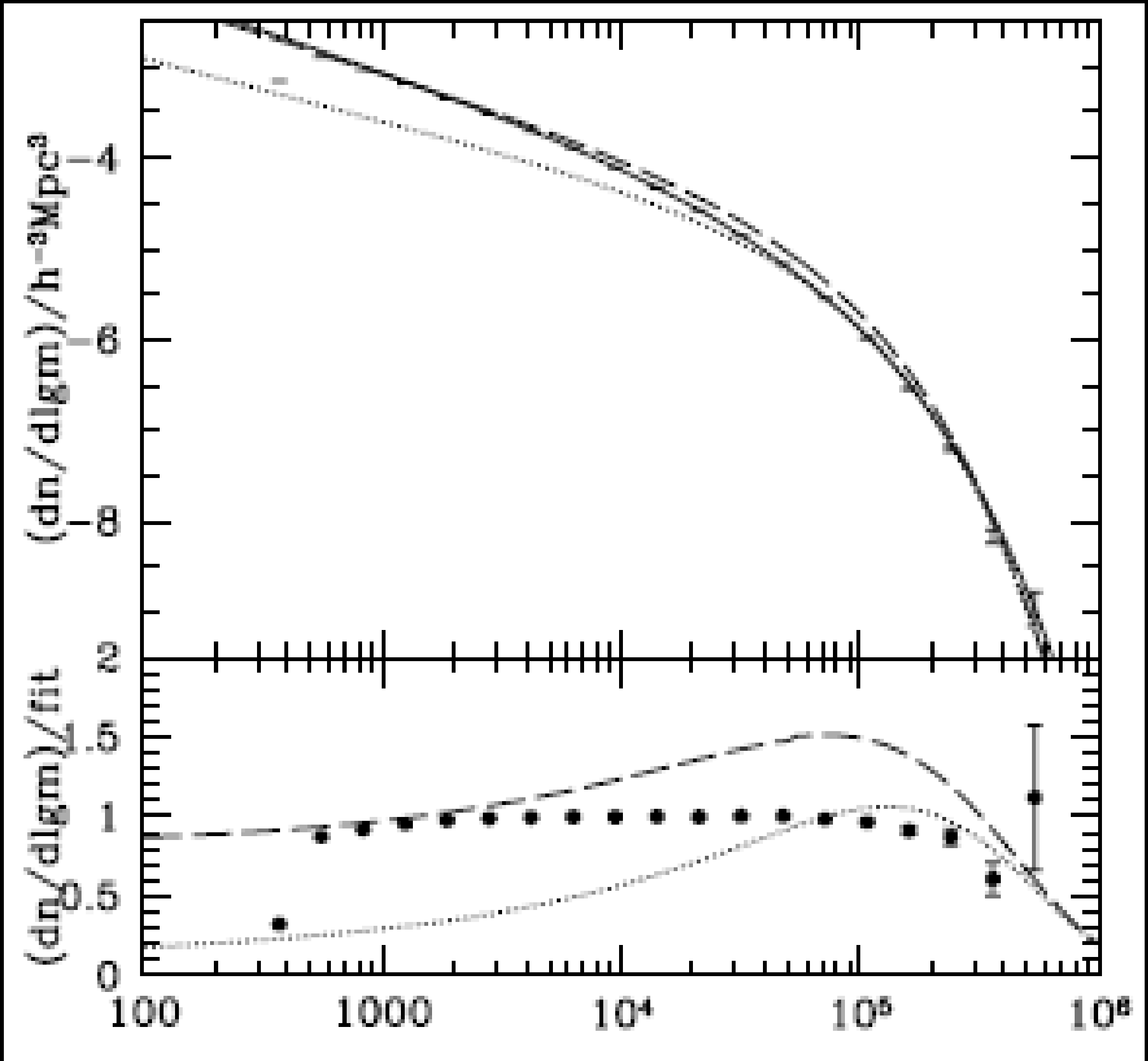


Halos
Doroshkevich + Peaks
Doroshkevich (1970)
p(x)
x = (eδ/σ)²
0
1
2
3
4
5
0
0.2
0.4
0.6
0.8
1

- Velocity shear field also like that for peaks (smaller variance than around field points) independent of mass



Peaks mass function OK at highest masses. Need better model for peaks-in-peaks



CAUTION:

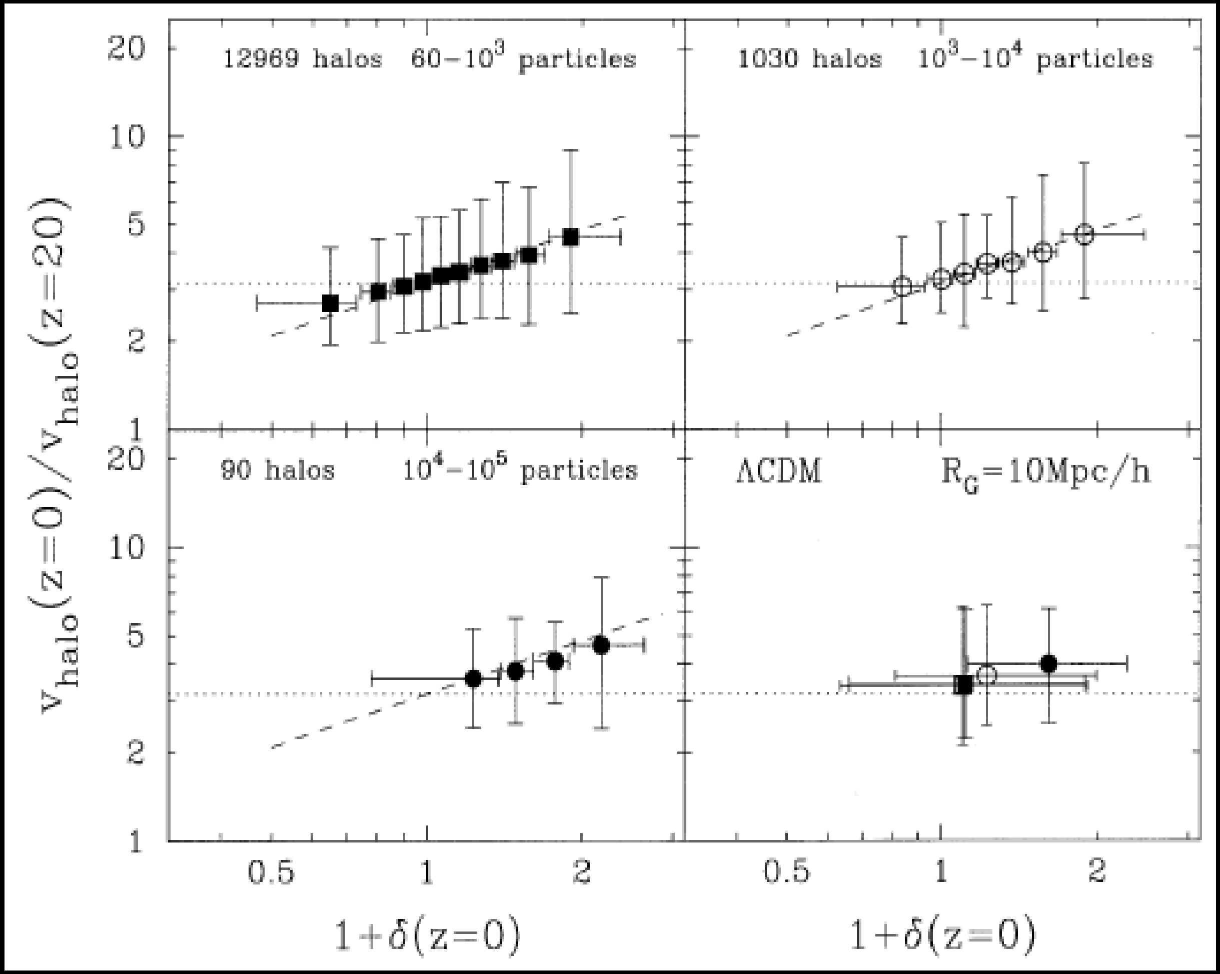
Velocity bias of peaks modifies Kaiser formula for z-space distortions

$$\delta_z = \delta_m \, [b_{pk}(k) + b_{vel}(k) \, f\mu^2]$$

More on z-space distortions

So, can have two samples, with same bias factor, but different environment

Velocity growth factor depends on environment



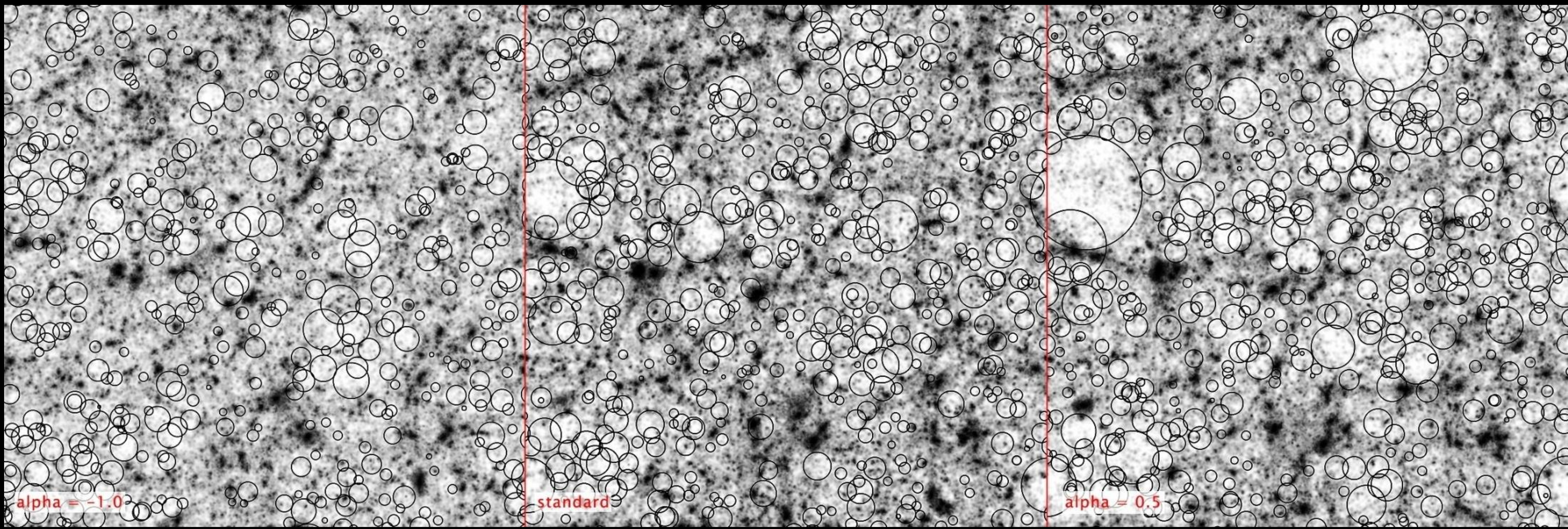
Sheth & Diaferio 2001

Summary

- Intimate connection between abundance and clustering of dark halos
  - Can use cluster clustering as check that cluster mass–observable relation correctly calibrated
  - Almost all correlations with environment arise through halo bias (massive halos populate densest regions)
- Next step in random walk ~ independent of previous
  - History of object ~ independent of environment
  - Not true for correlated steps
- Description quite detailed; sufficiently accurate to complete Neyman–Scott program (The Halo Model)

‘Modified’ gravity theories

Martino & Sheth 2009



weaker gravity on large scales stronger gravity

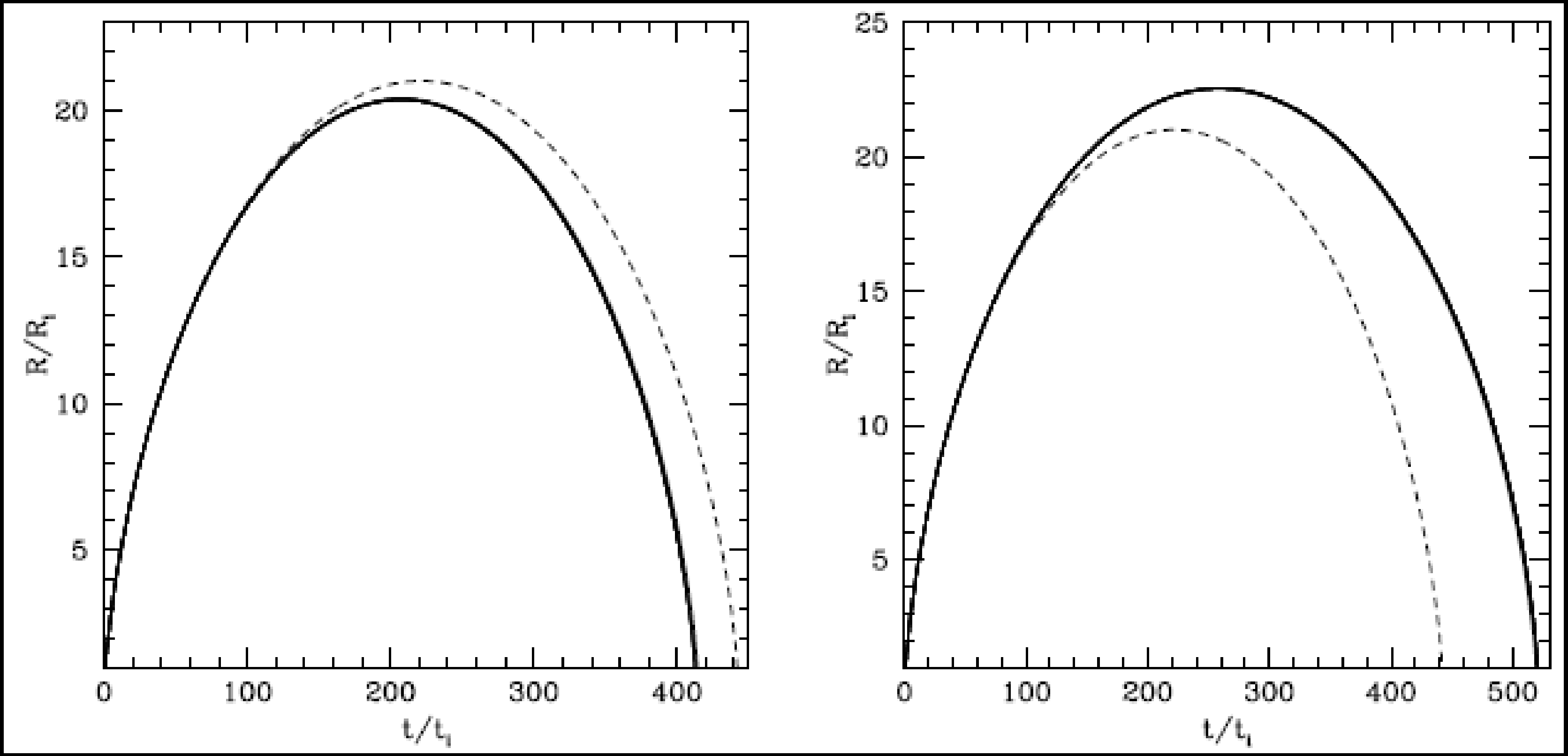
Voids/clusters/clustering are useful indicators

In many modifications to GR, scale-dependent linear theory is generic:

$$\delta(k,t) = D(k,t)\,\delta(k,t_i)$$

N.B. Peaks in ICs are not peaks in linearly evolved field

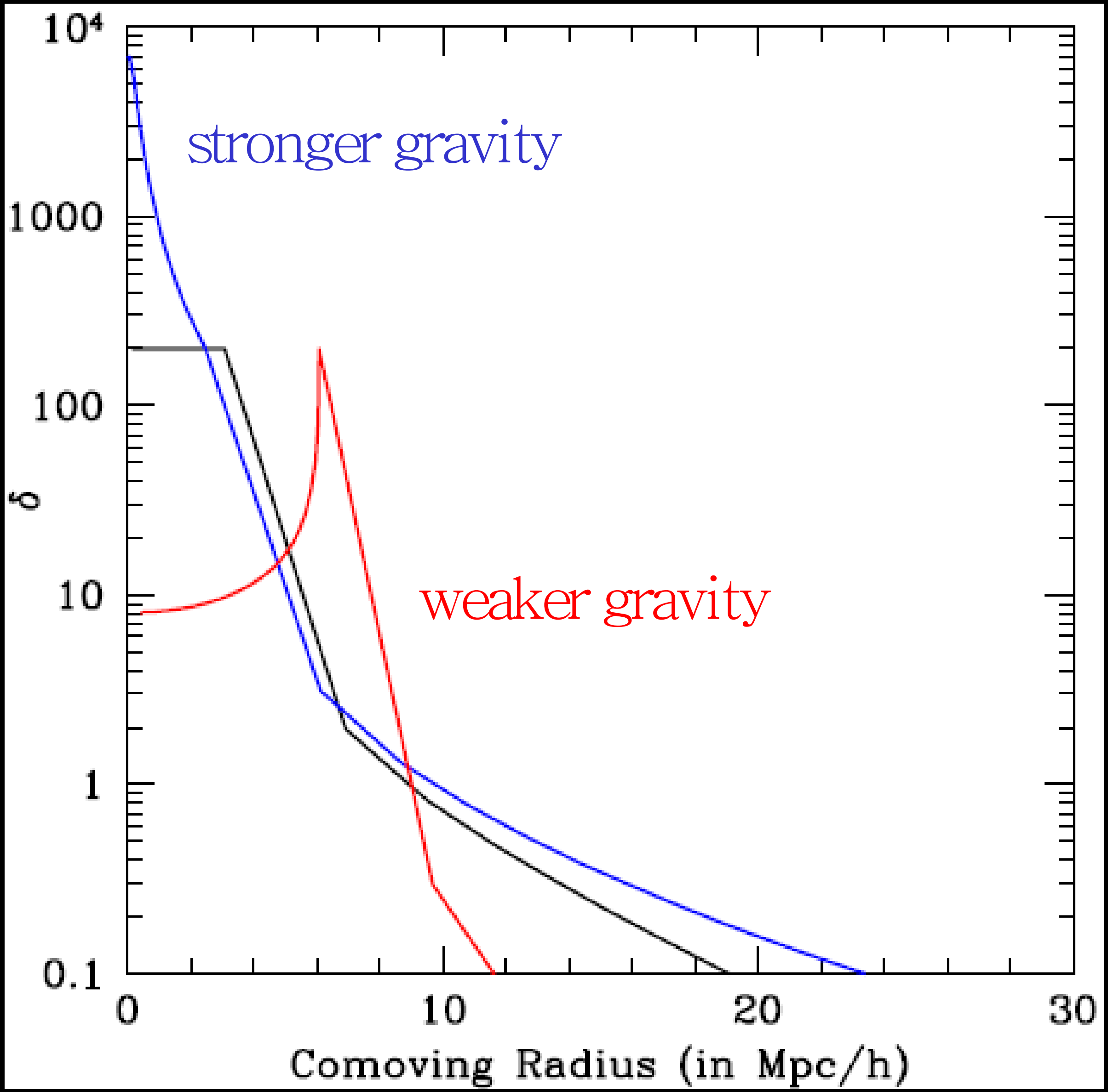
Modified force modifies (spherical) collapse



(Martino & Sheth 2009; Parfrey et al. 2011)

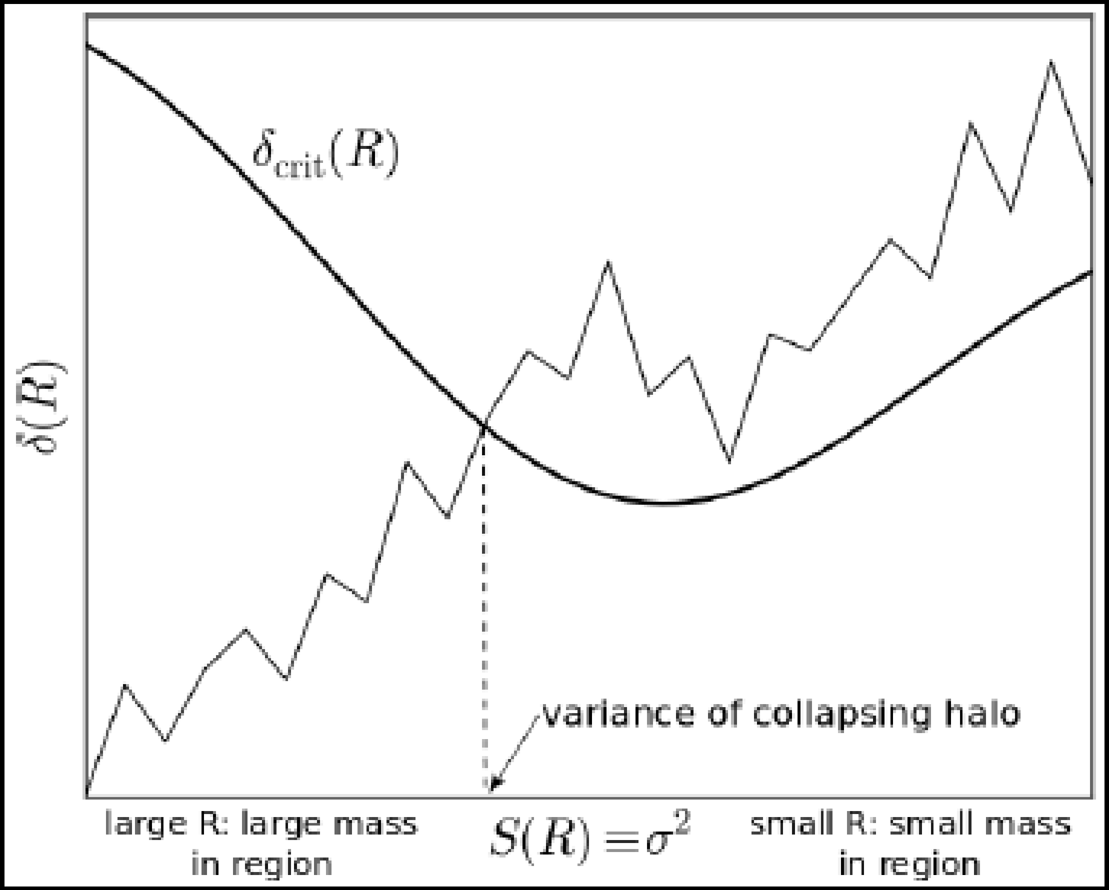
Halo density profiles in modified gravity theories:

X-ray observations useful?



Martino & Sheth 2009

Modified halo abundances …

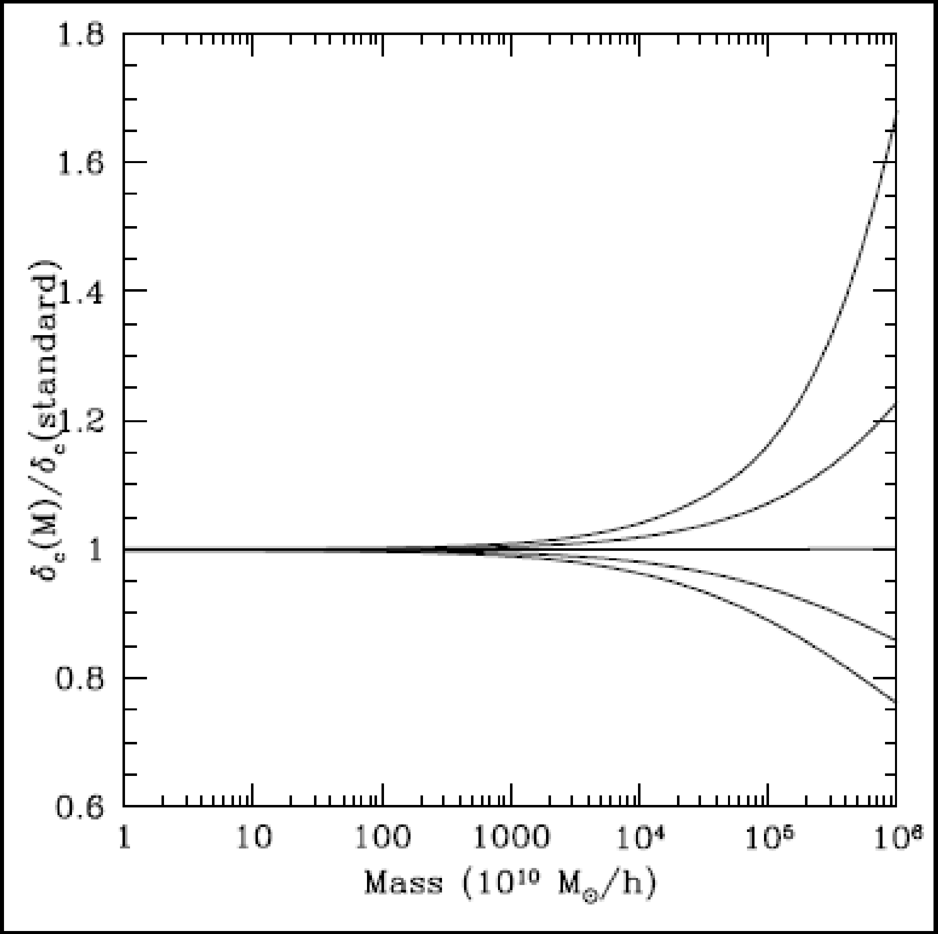


… potentially from two effects

$\delta_c(m)$ and $\sigma(m)$

- In standard GR, $\sigma(m)$ is an integral over initial P(k), evolved using linear theory to later (collapse) time.
- In modified theories, D(k,t) means there is a difference between calculating in ICs vs linearly evolved field.
- Energy conservation arguments suggest better to work in ICs.

Critical overdensity required for collapse at present becomes mass dependent: $\delta_c(m)$



This is generic.

Implication for halo bias

- Halos are linearly biased tracers of initial field
- D(k) means they are not linearly biased tracers of linearly-evolved field (Parfrey et al. 2011)

- Simulations in hand too small to test, but if real, this is an important systematic

Study of random walks with correlated steps

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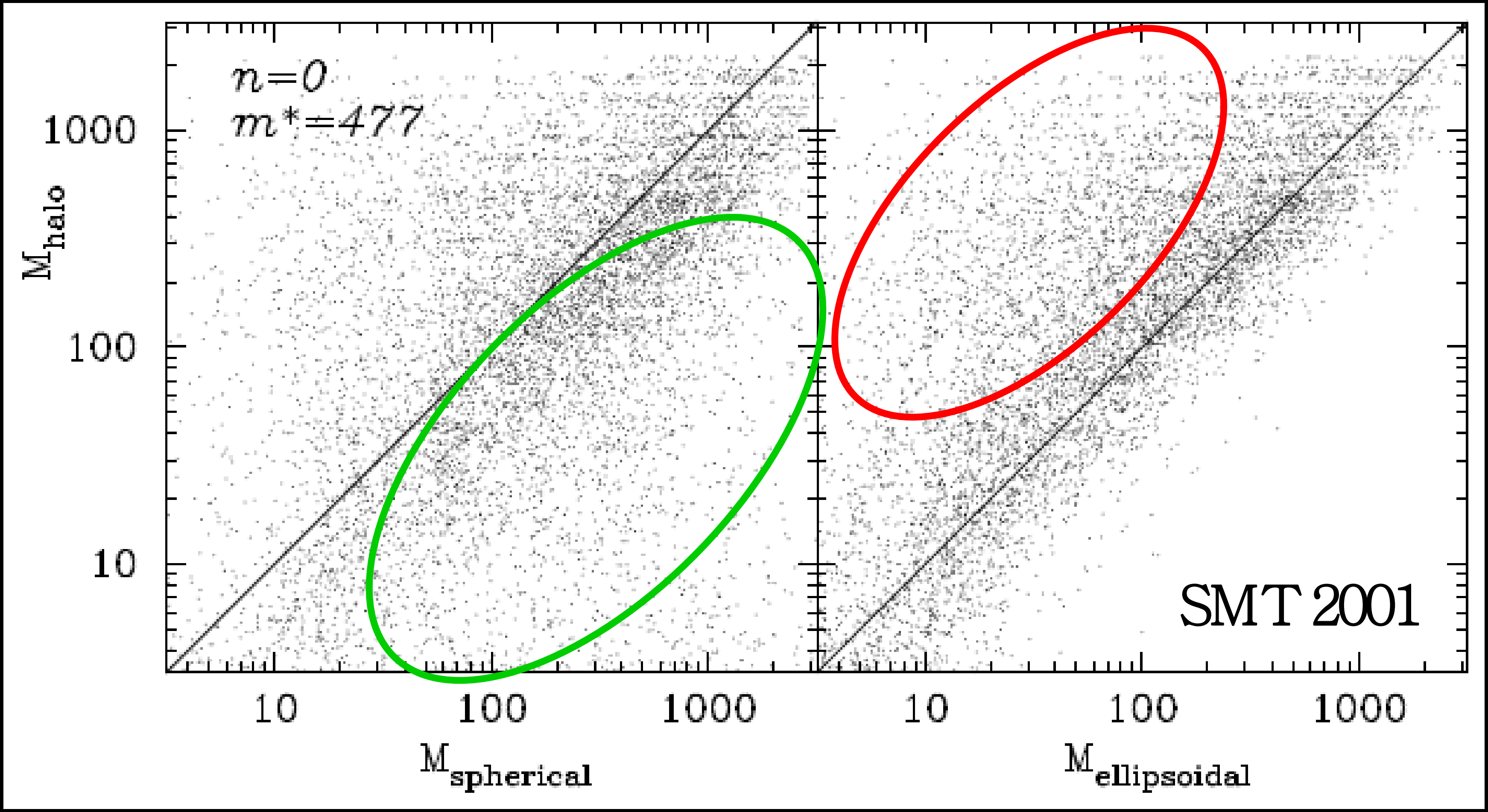
Cosmological constraints from large scale structures

Work in progress

- Correlated steps
  - Assembly bias
- Correlated walks
  - What is correct ensemble?
  - Generically increase high-m abundance (ST99 parameter 'a')
- Dependence on parameters other than δ
  - Higher dimensional walks
  - What is correct basis set? (e,p)? (I_2,I_3)?

The real cloud-in-cloud problem

Spherical collapse not full story



$M_{sim} > M_{predicted}$ often

- Write

$$f(m) = \int_m^{\infty} dM\, dn/dM\, (M/\rho)\, \phi(m|M)$$

where random walk $f(m) = f_{PS}(m,\delta_m)$ and ϕ is probability predicted mass is m given correct (simulated) mass is M

- Since M > m, model

$$\phi(m|M) \sim f_{PS}(m,\delta_m|M,\delta_M) \text{ with } \delta_m > \delta_M$$

random walk $f(m) = f_{PS}(m,\delta_m)$ means mass function $(M/\rho)\, dn/dM = f_{PS}(M,\delta_M)$

- Generically expect that accounting for this ~ decreasing δ_c

The real cloud-in-cloud problem

- Usual ansatz:

$$f(s)\,ds = f(m)\,dm = (m/\rho)\,dn/dm\;\;dm$$

- But using first crossing distribution only solves problem of concentric clouds
- A better ansatz?

$$f(m) = {}_m\!\int^{\infty} dM\; dn/dM\; (M/\rho)\; \phi(m|M)$$

where $\phi(m|M)$ is fraction of walks in M that first cross δ_c at m

- If

$$f(m) = \int_m^\infty dM \; dn/dM \; (M/\rho) \; \phi(m|M)$$

- Expect $\phi(m|M) \sim f_{PS}(m,\delta_m|M,\delta_M)$ with $\delta_m > \delta_M$
  - Can quantify difference because density run around 'peak' decreases with distance from peak
- Then random walk $f(m) = f_{PS}(m,\delta_m)$ means mass function $(M/\rho)\, dn/dM = f_{PS}(M|\delta_M)$
- Solving cloud-in-cloud problem ~ decreasing δ_c