Dark Energy Phenomenology

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Outline

- why go beyond a cosmological constant?
- geometry and effective fluid quantities
- some questions/examples
    - can we measure the geometry?
    - can we always find effective quantities?
    - scalar field ‘fluid’ and phantom crossing
    - anisotropic stress as GR modification diagnostic
- and some limitations
    - degeneracies
    - non-linearities

The Nobel Prize 2011

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

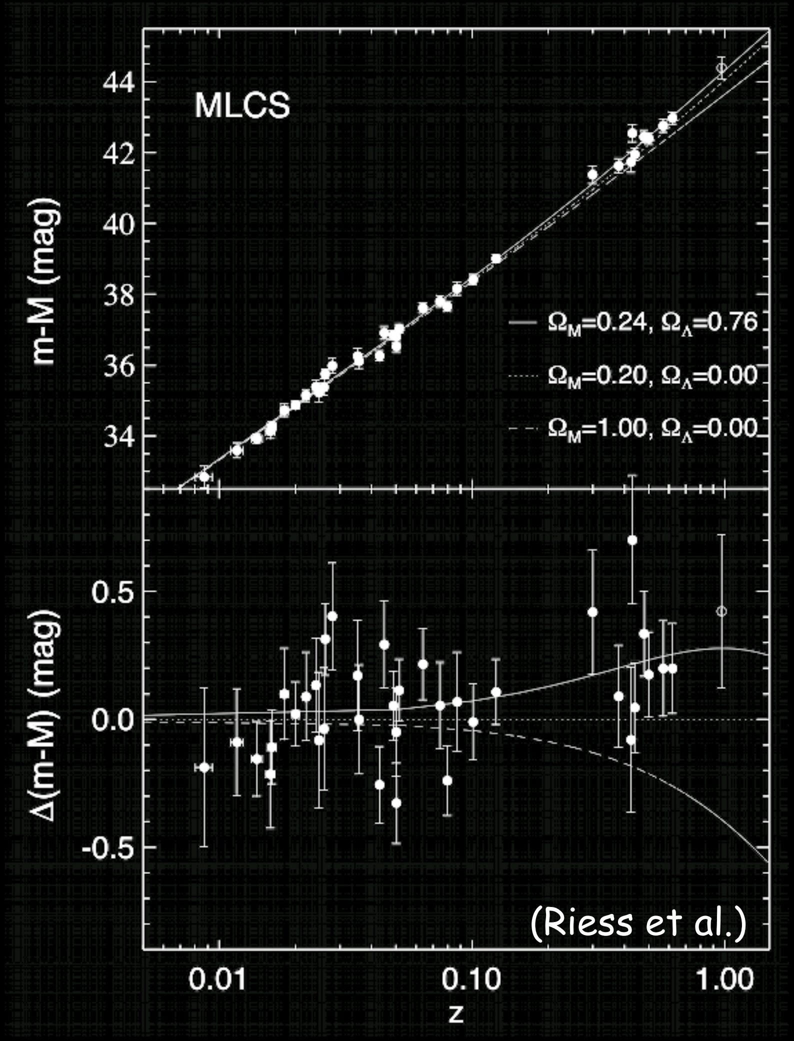
The Universe is now officially accelerating, thanks to the prize given to Saul Perlmutter, Brian P. Schmidt and Adam G. Riess, and we need to understand the reason!

One well-motivated model:

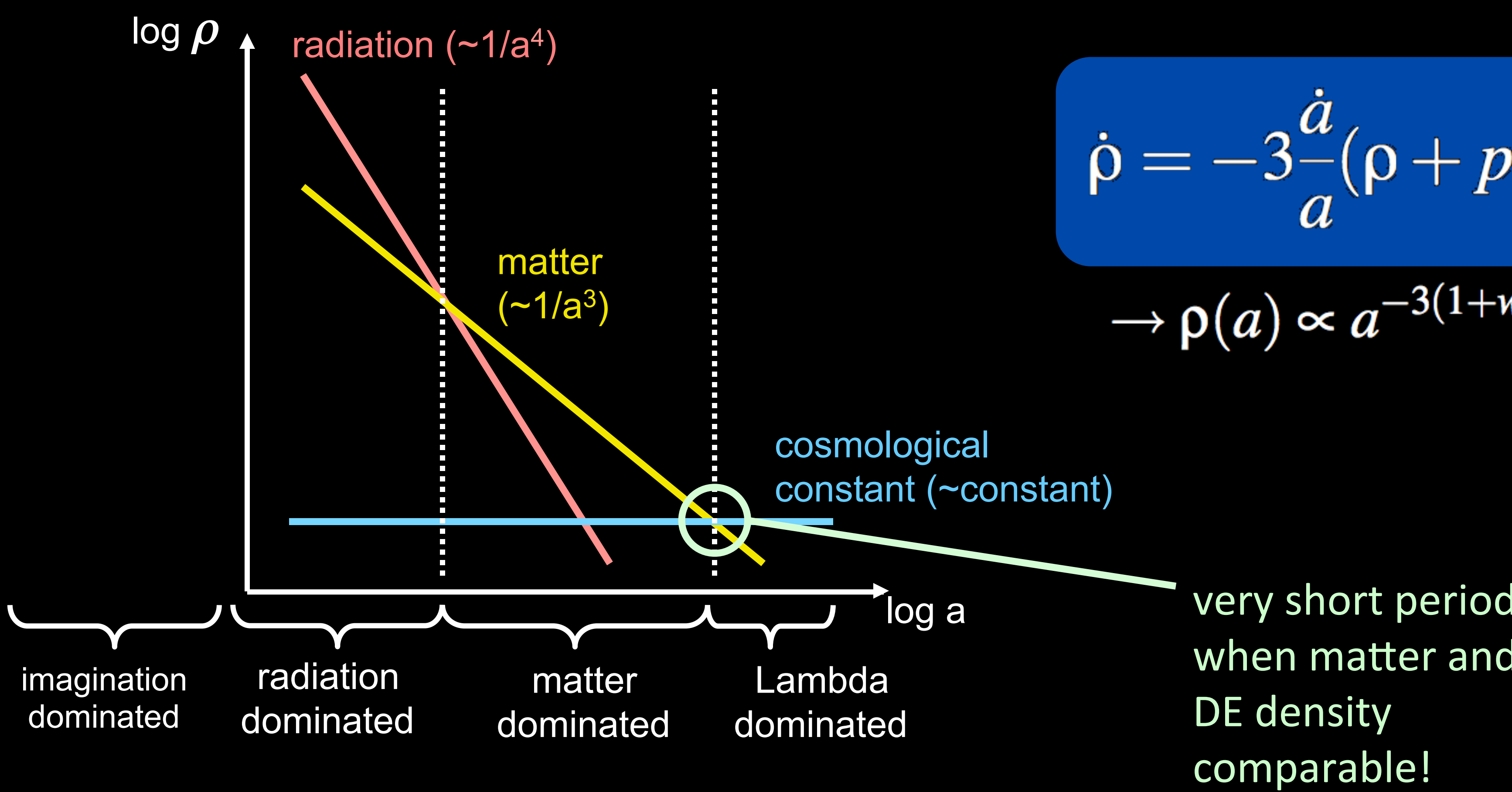
the cosmological constant

but (my personal list):

- coincidence problem
- size problem
- inflation was no cosmological constant



the coincidence problem



$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$

$$\rightarrow \rho(a) \propto a^{-3(1+w)}$$

size matters

energy scale of observed Λ is ~ 2×10^{-3} eV
zero point fluctuations of a heavier particle of mass m:

cutoff scale

$$\int_0^{\Lambda} \frac{1}{2} \frac{d^3p}{(2\pi)^3} \sqrt{p^2+m^2} \simeq \frac{1}{16\pi^2} \left[\underbrace{\Lambda^4 + m^2\Lambda^2}_{\text{can in principle be absorbed into renormalization of observables}} - \underbrace{\frac{1}{4} m^4 \log\left(\frac{\Lambda^2}{m^2}\right)}_{\text{``running'' term: this term is measureable for masses and couplings! Why not for cosmological constant?!}} \right]$$

already the electron should contribute at m_e >> eV
(and the muon, and all other known particles!)

(thanks to Andrew Tolley for long explanations!)

w during inflation

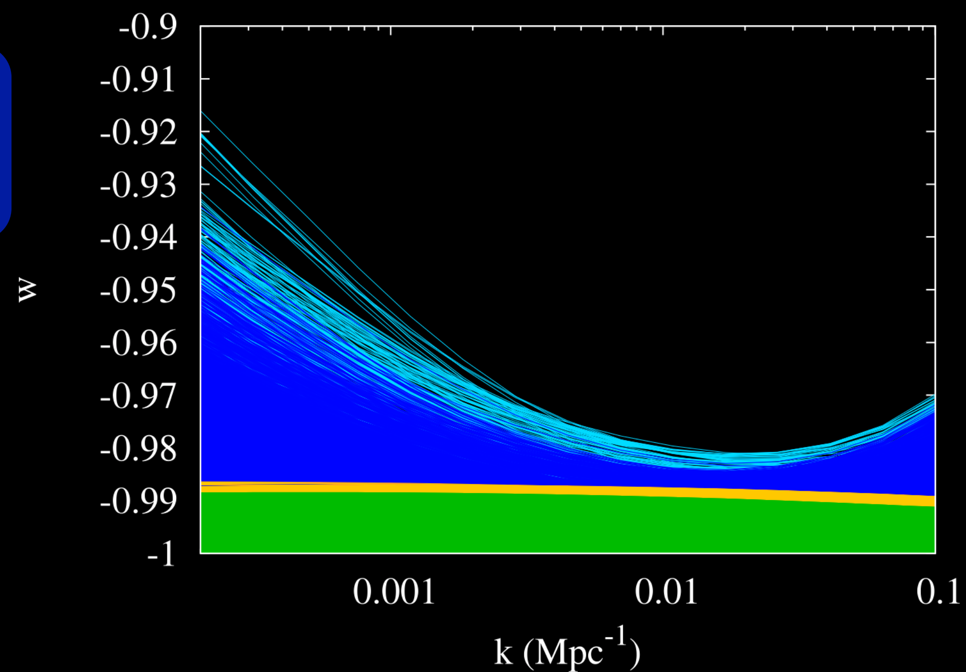
(Ilic, MK, Liddle & Frieman, 2010)

- Scalar field inflaton: $1+w = -\frac{2}{3}\frac{\dot{H}}{H^2} = \frac{2}{3}\epsilon_H$ and r = T/S ~ 24 (1+w)
- Link to dw/da: $\frac{d\ln(1+w)}{dN} = 2(\eta_H - \epsilon_H)$ $\quad 2\eta_H = (n_s - 1) + 4\epsilon_H$

$n_s \neq 1 \Rightarrow \varepsilon \neq 0$ or $\eta \neq 0$
=> w ≠ -1 and/or w not constant

WMAP 5yr constraints on w:
- (1+w) < 0.02
- No deviation from w=-1 necessary

(but not clear if representative of dark energy)



→ w ~ -1 appears natural during observable period of inflation
→ but it was not an (even effective) cosmological constant!

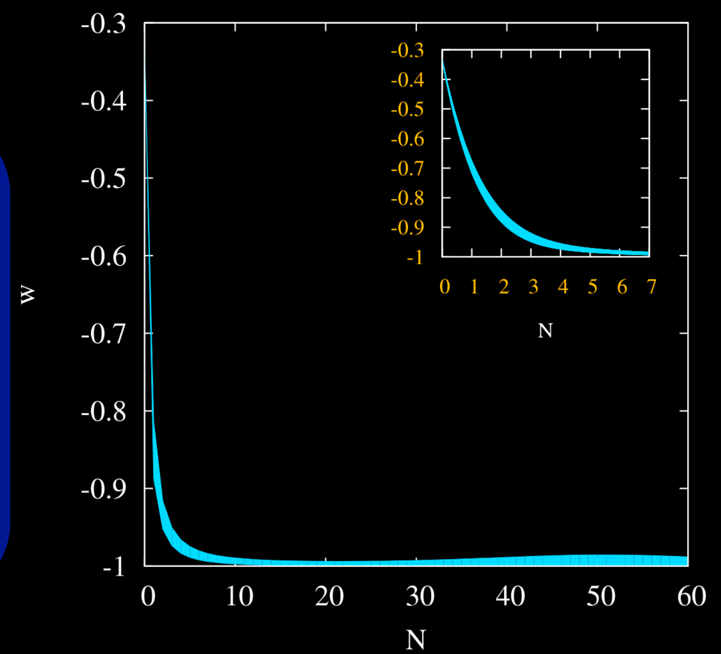
naïve expectations

$$2\eta_H = (n_s - 1) + 4\epsilon_H \quad \rightarrow 6\,(1+w) = -\,(n_s-1) + 2\,\eta_H \approx 0.04 + 2\,\eta_H$$

- η_H precisely canceling n_s-1 appears fine-tuned
- if ε_H and η_H are of a similar size, then (1+w) ~ 0.005 to 0.015
- need accuracy of O(1%) to observe this!
- but: there are inflation models where $\varepsilon_H \ll \eta_H$

On the other hand, looking at the dynamics in the middle of a long slow-roll phase of a scalar field is very pessimistic. Close to the end, there is much more dynamics!

Of course, maybe inflation and dark energy are not related at all.



but if not Λ then it is …

… quartessence, quintessence, quintom, cold dark energy, Chaplygin gas, Kardassian expansion, K-essence, DBI, DGP, scalar-tensor, f(R), f(G), f(R,G), TeVeS, galileons, kinetic gravity braiding, degravitation, branes, …

- what is the right model?
- what if we overlooked a model?
- what can we actually measure in cosmology?

-> invert the approach: start with effective quantities that can be measured, parameterize those, compare with predictions and map results back to 'model space' :

Dark Energy Phenomenology

measuring dark things
(in cosmology)

Einstein: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

(determined by the metric) geomet

stuff
(what is it?)

ur favourite theory

something
else

assuming FLRW:

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

$$\dot\rho = -3\frac{\dot a}{a}(\rho + p)$$

nologists observe the
metry of space time

is depends on the ***total***
energy momentum tensor

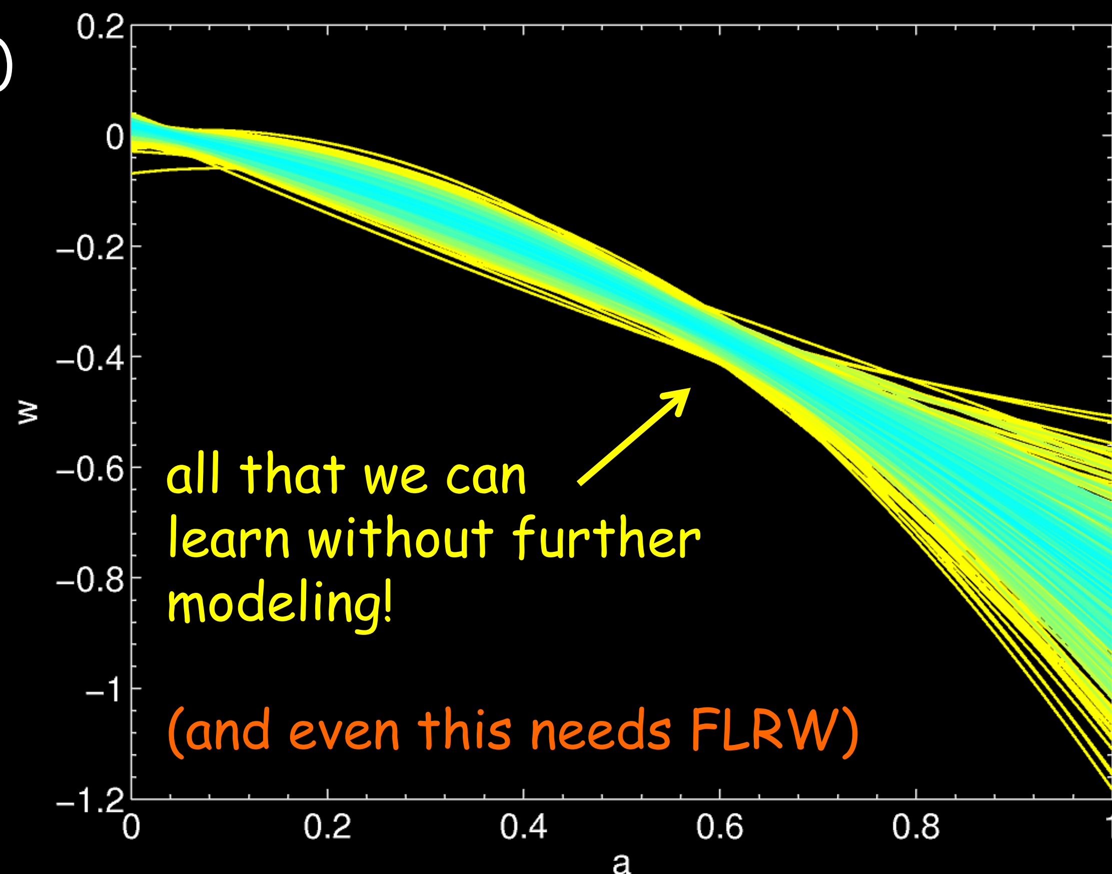
That is what we measure!

example: total w

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho \qquad \dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$

→ rewrite p = w ρ

- quadratic expansion of w(a)
- fit to Union SNe, BAO and CMB peak location
  → just distances, no perturbations
- best: χ^2 = 309.8
- ΛCDM: χ^2 = 311.9
- w const.: χ^2 = 391.3



MK, A. Liddle, D. Parkinson & C. Gao, PRD 80, 083533 (2009)

beyond the background

what can we learn? what do we need?

Einstein eq. (possibly effective):

$$G_{\mu\nu} - 8\pi G T_{\mu\nu}^{(bright)} = 8\pi G T_{\mu\nu}^{(dark)}$$

directly measured

given by metric:

- $H(z)$
- $\Phi(z,k)$, $\Psi(z,k)$

- inferred from lhs
- obeys conservation laws
- can be characterised by
  - $p = w(z)\,\rho$
  - $\delta p = c_s^2(z,k)\,\delta\rho$, $\pi(z,k)$

(MK & Sapone 2007;
Hu & Sawicki 2007;
Amendola, MK & Sapone 2008)

linear perturbation equations

metric: $$ds^2 = -(1+2\psi)dt^2 + a(t)^2(1-2\phi)dx^2$$

conservation equations (in principle for full dark sector)

$$\delta_i' = 3(1+w_i)\phi' - \frac{V_i}{Ha^2} - \frac{3}{a}\left(\frac{\delta p_i}{\rho_i} - w_i\delta_i\right) \qquad \delta p = c_s^2\delta\rho + 3Ha(c_s^2 - c_a^2)\rho\frac{V}{k^2}$$

$$V_i' = -(1-3w_i)\frac{V_i}{a} + \frac{k^2}{Ha}\left(\frac{\delta p_i}{\rho_i} + (1+w_i)(\psi - \sigma_i)\right)$$

(vars: $\delta=\delta\rho/\rho$, V ~ divergence of velocity field, δp , σ anisotropic stress)

Einstein equations (common, may be modified if not GR)

$$k^2\phi = -4\pi G a^2 \sum_i \rho_i \left(\delta_i + 3Ha\frac{V_i}{k^2}\right)$$

$$k^2(\phi - \psi) = 12\pi G a^2 \sum_i (1+w_i)\rho_i\sigma_i$$

(Bardeen 1980)

parametrising the dark side

small perturbations: extended metric

$$ds^2 = -(1+2\psi)dt^2 + a(t)^2(1-2\phi)dx^2$$

ϕ, ψ gravitational potentials <-> $\delta\rho$ and V perturbations of $T^{\mu\nu}$

Einstein and conserv. eqs.

$\longrightarrow$ fluid properties

$\delta p = c_s^2\, \delta\rho$ in DE rest frame

σ (anisotropic stress, $\phi = \psi$ for σ=0)

Alternatively: $k^2\phi = -4\pi G a^2 Q \rho_m \Delta_m \qquad \psi = (1+\eta)\phi$

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background:	H	↔	w		
perturbations:	Φ Ψ	↔	Q η	↔	δp / c_s^2 σ / π

in principle all equivalent

check-point

-> measure potentials, infer effective fluid properties, understand dark sector physics

- can we measure the potentials?
- can we always find a phenomenological description?
- what are these quantities: some predictions
- quintessence example
- can we map physics backwards? (π example)
- what are the limits?

simplified observations

(how to measure these things?)

- Curvature from radial & transverse BAO
- $w(z)$ from SN-Ia, BAO directly (and contained in most other probes)
- In addition 5 quantities, e.g. ϕ, ψ, bias, δ_m, V_m
- Need 3 probes (since 2 cons eq for DM)
- e.g. 3 power spectra: lensing, galaxy, velocity
- Lensing probes $\phi + \psi$
- Velocity probes ψ (z-space distortions?)
- And galaxy P(k) then gives bias

→ *Euclid* can do it all

the geometric EMT

(G. Ballesteros, L. Hollenstein, R. Jain & MK)

$$1 + w_G = -\frac{2}{3}\frac{\dot{H}}{H^2}$$

$$\delta\rho_G = -2M_P^2\left[3H\left(\dot{\phi} + H\psi\right) - a^{-2}\nabla^2\phi\right]$$

$$\delta p_G = 2M_P^2\left[\ddot{\phi} + H\left(3\dot{\phi} + \dot{\psi}\right) - 3w_G H^2\psi - \frac{1}{3}a^{-2}\nabla^2\Pi\right]$$

$$\delta q_{\mu\,G} = -2M_P^2\,\delta_\mu^i\left[\partial_i\left(\dot{\phi} + H\psi\right)\right]$$

$$\delta\pi_{\mu\nu\,G} = M_P^2\,\delta_\mu^i\delta_\nu^j\left[\left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\Pi\right]$$

$$\Pi = \phi - \psi$$

We can always reconstruct an effective fluid EMT that gives the observed metric!

some model predictions

$$\boxed{k^2\phi = -4\pi G a^2 Q \rho_m \Delta_m \qquad \psi = (1+\eta)\phi}$$

scalar field: $$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi)\right)$$

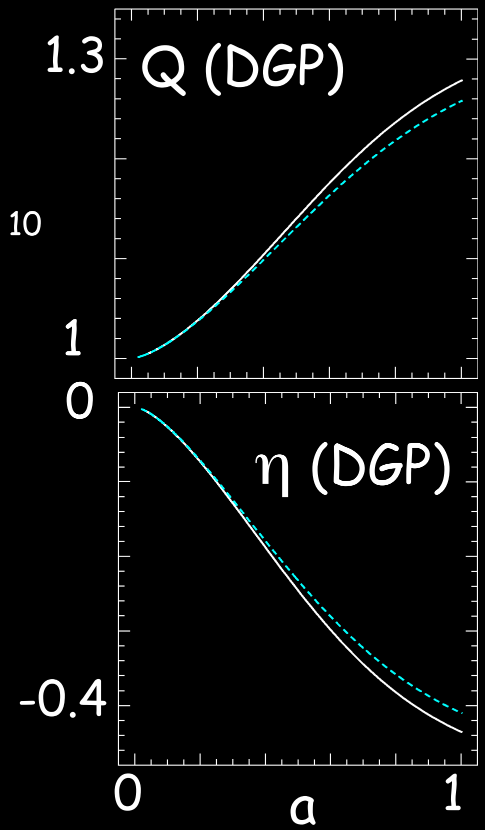
One degree of freedom: $V(\phi)$ <-> $w(z)$
therefore other variables fixed: $c_s^2 = 1$, $\pi = 0$
-> $\eta = 0$, $Q(k \gg H_0) = 1$, $Q(k \sim H_0) \sim 1.1$ cf Sapone, MK 09
Sapone, Amendola, MK 10

(naïve) DGP: compute in 5D, project result to 4D

Lue, Starkmann 04
Koyama, Maartens 06
Hu, Sawicki 07

$$\eta = \frac{2}{3\beta - 1} \qquad Q = 1 - \frac{1}{3\beta}$$

implies large DE perturb.



Scalar-Tensor: Boisseau, Esposito-Farese, Polarski, Starobinski 2000,
Acquaviva, Baccigalupi, Perrotta 04

$$\mathcal{L} = F(\varphi)R - \partial_\mu\varphi\partial^\mu\varphi - 2V(\phi) + 16\pi G^* \mathcal{L}_{\rm matter}$$

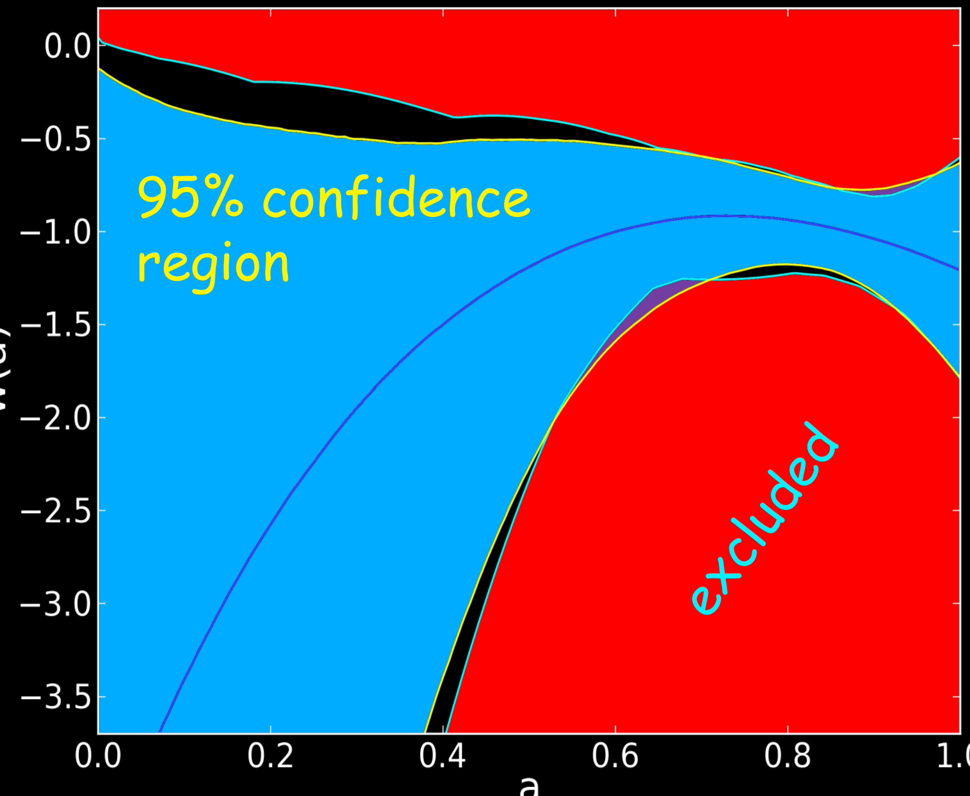
$$\eta = \frac{F'^2}{F + F'^2} \qquad Q = \frac{G^*}{F G_0}\frac{2(F + F'^2)}{2F + 3F'^2}$$

Quintessence example

canonical scalar field model ↔ $c_s^2=1$, $\pi=0$

- use effective fluid model defined as above for DE
- WMAP-7yr + SN-Ia compilation
- regularised transition of w=-1
- cubic expansion of w(a)

- cosmological constant fits well
- $|1+w| < 0.2$ at a ~ 0.8 @ 2σ
- constraints only valid for this model!



(based on work esp. w/ Bassett, Corasaniti & Parkinson)

easy to extend to K-essence by allowing for different c_s^2
(but how did we actually cross w=-1?)

phantom crossing

(e.g. MK & D. Sapone 2006)

apparent issue: $$\dot{\theta} = -\frac{\dot{w}}{1+w}\theta + \ldots$$

due to parametrizing $T_0{}^i{}_{,i} \sim (\rho+p)\,\theta$
easily remedied by setting $V = (1+w)\,\theta$ for the velocity.

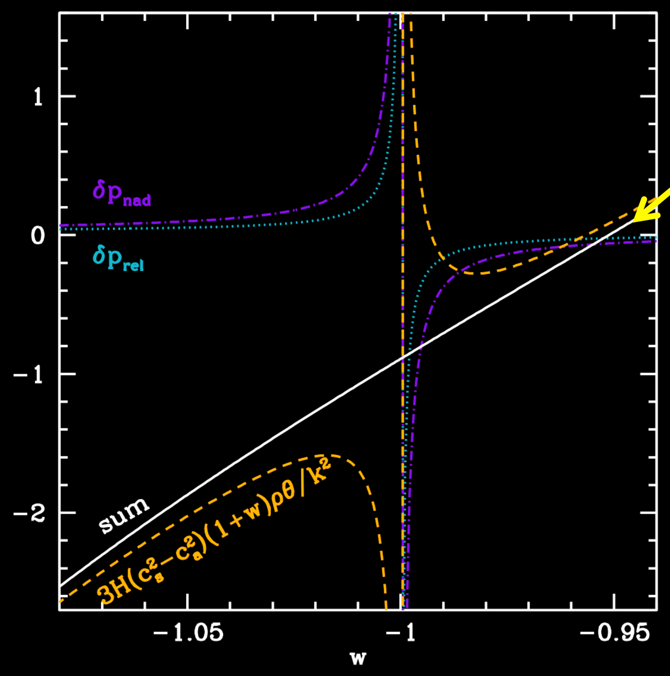
more serious issue: we want to set $\delta p = c_s^2\,\delta\rho$ in fluid rest frame
-> gauge transformation to other frame:

$$\delta p = c_s^2 \delta\rho + 3\mathcal{H}(c_s^2 - c_a^2)\rho\frac{V}{k^2} \qquad c_a^2 = \frac{\dot{p}}{\dot{\rho}} = w - \frac{\dot{w}}{3\mathcal{H}(1+w)}$$

this transformation blows up (there is no DE rest frame for w=-1),
except if V -> 0 fast enough <-> $w'=0$ or $c_s^2=0$ at crossing

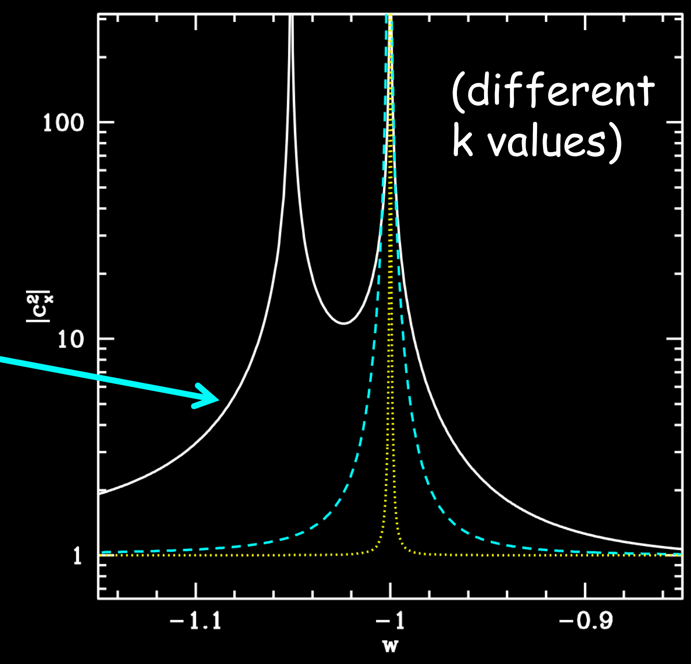
quintom crossing

But it is possible to cross without these conditions!
E.g. quintom: 2 fluids/fields with $w_1 > -1$ and $w_2 < -1$



total δp is finite

eff. c_s^2 is not!



- In principle all parameterizations are equivalent.
- In practice less so…
- But the EMT is the fundamental object!

fundamental cosmology?

(with L. Amendola, I. Saltas & I. Sawicki)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}(1+f(\phi))R + K(\phi,X) + \mathcal{L}_\mathrm{b}[g_{\mu\nu}] + e^{2c(\phi)}\mathcal{L}_m[g_{\mu\nu}e^{c(\phi)}]\right]$$

1. radiation, baryons, dark matter + dark energy
2. baryons and dark matter are pressureless
3. we know Ω_b

   baryons move on geodesics
4. radiation moves on geodesics

   average metric coincides with FLRW metric
6. DE is a single scalar field

7a DE unclustered on small scales -> K (+others)

7b DE has no anisotropic stress -> f

7c DE-DM are uncoupled -> c

(8 combinations of the above)

the importance of η / σ / π

(MK, Sapone 2007; Amendola, MK, Sapone 2008; Saltas & MK 2011)

in the previous model: $\pi \sim \frac{f'}{1+f}\delta\phi$

-> unique link π <-> MG

quintessence, K-essence, etc: $\pi = 0$

DGP, S/T, f(R), f(G), etc: $\pi \neq 0$

(except in GR limit)

-> extra scalar d.o.f. very directly linked to π

→ η can rule out whole classes of models!

Can we break this conclusion? I.e. how easy is it to find a MG model with no effective anisotropic stress?

How robust is anisotropic stress as a MG signature?

(Ippocratis Saltas & MK 2011)

DGP: $\eta = \frac{2}{3\beta - 1}$ $\quad \beta = 1 + 2 r_c H w_{\mathrm{DE}}$

$$\eta \to 0 \Rightarrow \beta \to \infty \Rightarrow r_c = \frac{M_4^2}{M_5^3} \to \infty$$

-> effectively only 4D EH action left

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scalar/ tensor: $\mathcal{L} = F(\varphi)R - \partial_\mu \varphi \partial^\mu \varphi - 2V(\phi) + 16\pi G^* \mathcal{L}_{\mathrm{matter}}$

$\eta = \frac{F'^2}{F + F'^2}$ $\quad \eta \to 0 \Rightarrow F'(\varphi) \to 0$

-> F const -> uncoupled EH action left

MG anisotropic stress

f(R):
(e.g. De Felice & Tsujikawa Living Reviews)

$$S_g \sim \int d^4x \sqrt{-g} f(R) \qquad \phi - \psi = \frac{f_{RR}}{f_R} \delta R$$

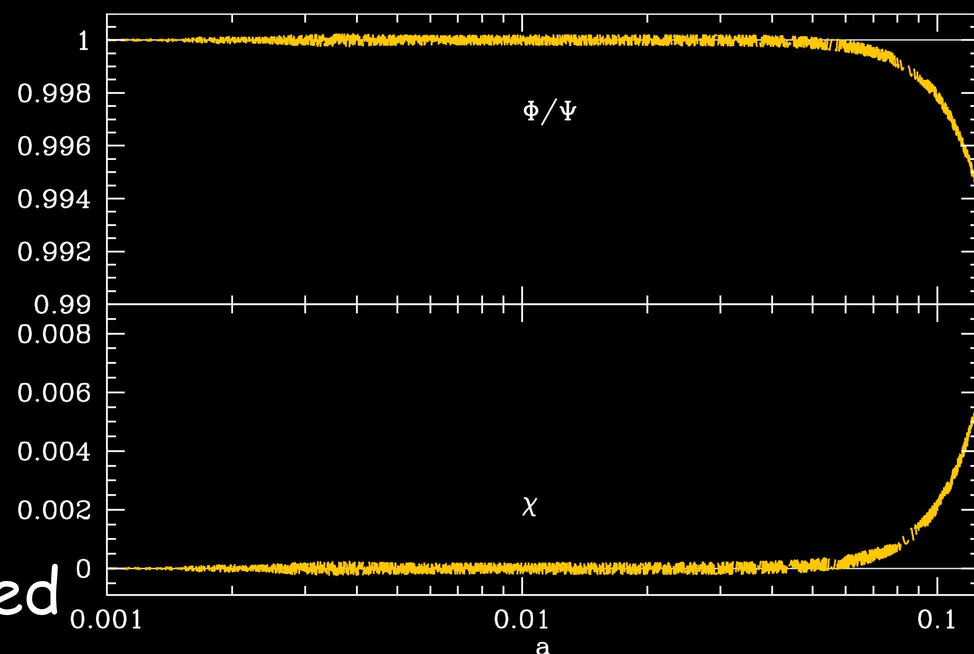
$$\eta = 0 \Rightarrow f_{RR} = 0 \Rightarrow f_R \text{ const} \Rightarrow f(R) \propto R + \Lambda$$

-> only GR allowed

extra d.o.f.:
"scalaron" with mass

$$m_{\text{eff}}^2 \propto \frac{f_R}{f_{RR}}$$

doesn't like to be suppressed



(Pogosian & Silvestri 2008)

MG anisotropic stress

f(G):

(De Felice & Suyama 2009)

$$S_g \sim \int d^4x \sqrt{-g}\big(R + f(G)\big)$$

$$G = R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$$

(beware of Ostrogradski instability for other combinations)

$$\phi - \psi = 4H^2 f_{GG}\delta G$$

$$\phi = \psi \Rightarrow f(G) = G + \Lambda$$

-> again GR in 4D

this time the scalaron has a mass $m^2_{\rm eff} \propto \dfrac{1}{H^4 f_{GG}}$

MG anisotropic stress

f(R,G): $$S_g \sim \int d^4x \sqrt{-g} f(R,G)$$

-> now can in principle balance R & G contribution to anisotropic stress, e.g. for de Sitter:

$$f_{RR} + 8H_0^2 f_{RG} + 16H_0^4 f_{GG} = 0$$

general solution

$$f(R,G) = f_1[R - G/(4M^2)] + R f_2[R - G/(4M^2)]$$

for $M=H_0$ - but:

- scalaron mass again diverges at this point
- solution is unstable for small perturbations δH
- sound speed c_s -> ∞

limits of DE phenomenology

- degeneracies
- non-linearities
- environment-dependent effects:

  viable MG theories need to suppress extra d.o.f. in very dense regions (e.g. solar system), so effective parameters of DE change as function of environment. (But is this really different from scale-dependence, and if so, doesn't this affect all analyses that assume statistical isotropy?)

degeneracies

- we can always reconstruct the total EMT
- we expect to have at least DM + DE, maybe with couplings
- but we cannot see 'inside' the EMT!

example: assume DM+DE then DE equation of state

$$w(z) = \frac{H(z)^2 - \frac{2}{3}H(z)H'(z)(1+z)}{\Omega_m H_0^2(1+z)^3 - H(z)^2}$$

for any choice of Ω_m (not only right one)!

-> we can generate families of possible 'DE' parameters, but only theoretically motivated models (or non-gravitational tests) can favour some of those.

nonlinearities

(G. Ballesteros, L. Hollenstein, R. Jain & MK, preliminary)

key questions:

- is the pressure perturbation nonzero?
  -> rules out ΛCDM
- is the anisotropic stress nonzero? (beyond γ/ν)
  -> probably modified gravity model

but only simple at linear perturbation level!

Even in ΛCDM velocities induce a (observer dependent) pressure and anisotropic stress at second order!

transverse trace-free part

$$p_G = \frac{1}{3}\rho_m v_m^2 \, , \qquad \pi_G^{\mu\nu} = \rho_m v_m^{\langle\mu} v_m^{\nu\rangle}$$

-> geometric quantities need careful interpretation and corrections esp. in non-linear regime!

Conclusions

- testing ΛCDM is a key goal
- but every model is limited, so a general approach is important
- the phenomenological quantities allow also for general conclusions (e.g. π)
- can try to map parameters back into model / action space
- but challenges and degeneracies remain