Cyclically deformed defects

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Outline

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Introduction

$$L(x,t) = \frac{1}{2}(\phi')^2 + U(\phi)$$

The energy density of the field is given by two portions - gradient and potential. The topological mass, or total energy, is obtained through the integration of the energy density.

$$\rho = \frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 + U(\phi) \longrightarrow M = \int_{-\infty}^{+\infty} \rho(x)dx$$

The energy density can be written like: $\rho = \left(\frac{d\phi}{dx}\right)^2$

We have used a first order formalism, which guarantees a simple way to calculate the total energy.

$$\left.\begin{aligned} U(\phi) &= \frac{1}{2}\left(\frac{dW}{d\phi}\right)^2 = \frac{1}{2}W_\phi^2 \\ \frac{d\phi}{dx} &= W_\phi \end{aligned}\right\} \quad \begin{aligned} U &= \frac{1}{2}W_\phi^2 \\ \rho &= W_\phi^2 \end{aligned}$$

Introduction

With this new function we can calculate the energy in the following way:

$$M = E = \int_{-\infty}^{\infty} \left(\frac{d\phi}{dx}\right)^2 dx = \int_{-\infty}^{\infty} \left(\frac{d\phi}{dx}\right)\left(\frac{dW}{d\phi}\right) dx = \int_{-\infty}^{\infty} dW$$

$$E = W(\phi(-\infty)) - W(\phi(+\infty))$$

Potential features

$U(\phi)$ creates a set of points $\{\bar{\phi}_{1,}\,\dots,\bar{\phi}_{n,}\}$ such that $\frac{dU}{d\phi} = 0$ for $\phi = \bar{\phi}_i$ with $i = 1,2,\dots,n$.

The energy density is also minimized when $\phi(x,t)$ is constant and assumes one of the $\bar{\phi}_i(x)$.

$$\rho = \left(\frac{d\phi}{dx}\right)^2 \Longrightarrow \lim_{x\to\pm\infty} \frac{d\phi}{dx} \to 0$$

Introduction

If $U(\phi)$ has

- a single minimum $\phi = \bar{\phi}_i$, then
the solution must satisfy $\phi \to \bar{\phi}_i$ when $x \to \pm\infty$;

- degenerate minimum, then
$\phi(x)$ must tend to one of the $\bar{\phi}_i(x)$ when $x \to +\infty$ and to the same point or another one when $x \to -\infty$.

With this we can define the topological charge

$$Q = \phi(x) \to +\infty - \phi(x) \to -\infty$$

Defects are classified as topological (or kink-like) if they have topological charge. Otherwise, they are non-topological (or lump-like).

Introduction

Deformation triggered by Sine Gordon's solution

- Potential: $T(\chi) = 1 - \cos(\chi)$
- Solution: $\chi(x) = \pm 4arctnh(e^{x}) - \pi$
- Mass: $M = \int_{-\infty}^{\infty} \rho(x)dx = 8$

Deformation mechanism

Deformation process

$$w_\chi = \frac{d\chi}{dx} = y_\phi \chi_\phi = z_\psi \chi_\psi,$$
$$y_\phi = \frac{d\phi}{dx} = z_\psi \phi_\psi = w_\chi \phi_\chi,$$
$$z_\psi = \frac{d\psi}{dx} = w_\chi \psi_\chi = y_\phi \psi_\phi,$$

$$\frac{d\phi}{dx}\frac{d\chi}{d\phi} = \frac{d\chi}{dx}$$

$$\frac{d\psi}{dx}\frac{d\chi}{d\psi} = \frac{d\chi}{dx}$$

$$\alpha_\beta = 1/\beta_\alpha$$

- Functions that generates new families of defects.

$$\alpha, \beta = \varphi, \phi, \psi$$

  - Potentials:

$$T(\chi) = \frac{1}{2}w_\chi^2 \rightleftharpoons V(\phi) = \frac{1}{2}y_\phi^2 \rightleftharpoons W(\psi) = \frac{1}{2}z_\psi^2 \rightleftharpoons T(\chi) = \frac{1}{2}w_\chi^2$$

$$\rho_\phi = {y_\phi}^2 \qquad \rho_\psi = {z_\psi}^2$$

Deformation mechanism

Hyperbolic

- Original defect → $\chi(x) = \pm 4 arctnh(e^{x})$
- Deformation functions:

$$\phi_{\chi}^{(n)} = sech(n\,\chi) \qquad y_{\phi} = \omega_{\chi} sech[\chi - (n-1)\pi]$$

$$\psi_{\chi}^{(n)} = \tanh(n\,\chi) \qquad z_{\psi} = -\omega_{\chi} \tanh[\chi - (n-1)\pi]$$

- Constraint for the masses:

$$\begin{aligned} w_{\chi}^{2} &= w_{\chi}^{2}\left[\tanh(n\,\chi)^{2} + \mathrm{sech}(n\,\chi)^{2}\right] \\ &= w_{\chi}^{2}\left[\psi_{\chi}^{(n)\,2} + \phi_{\chi}^{(n)\,2}\right] \\ &= z_{\psi}^{2} + y_{\phi}^{2}, \end{aligned}$$

Deformation mechanism

Trigonometric

- Original defect $\longrightarrow$ $\chi(x) = \pm 4arctnh(e^{x})$

- Deformation functions:

$$\phi_{\chi}^{(n)} = \cos\left(\chi - (n-1)\pi\right) \qquad y_{\phi} = \omega_{\chi}\cos[\chi - (n-1)\pi],$$

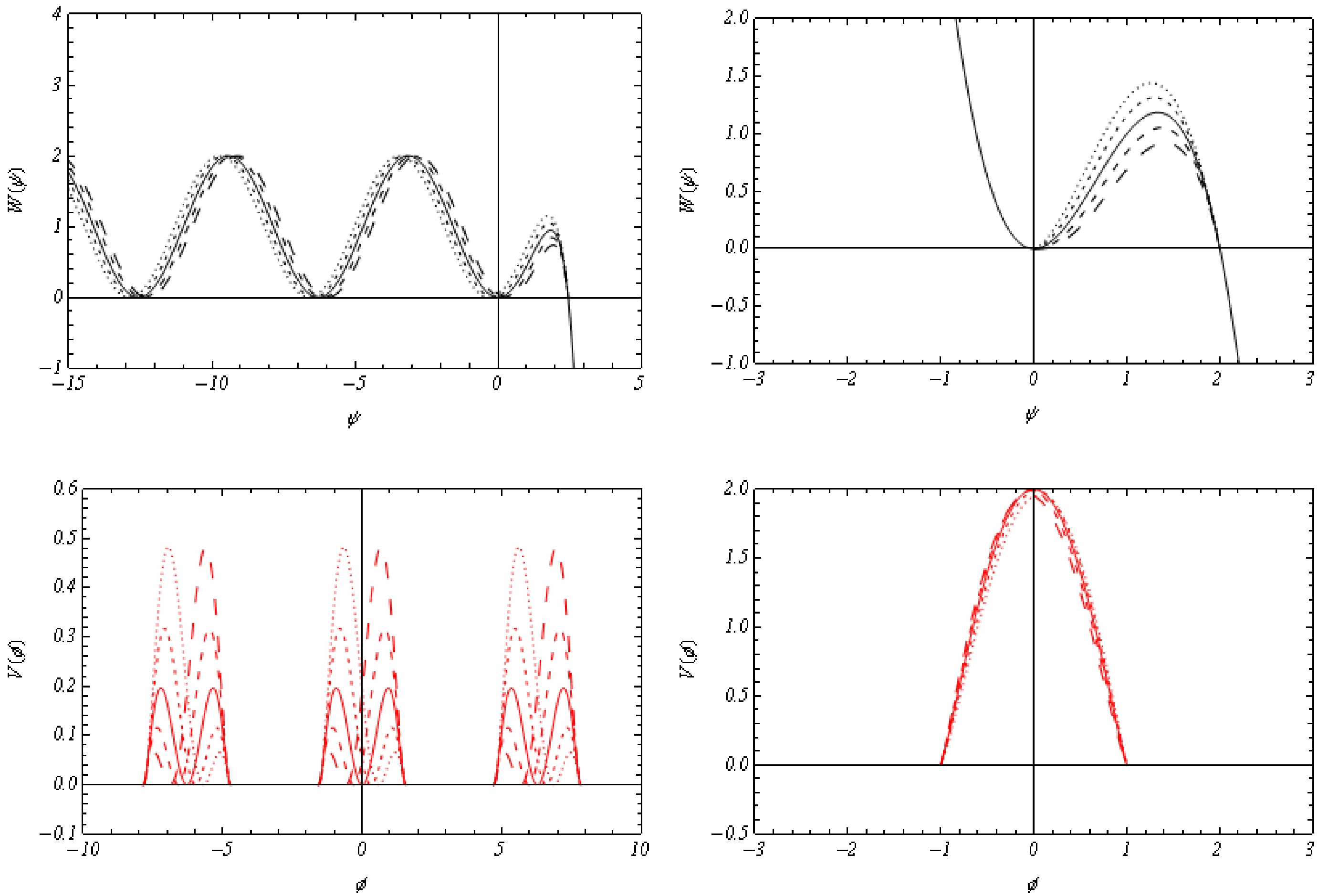
$$\psi_{\chi}^{(n)} = -\sin\left(\chi - (n-1)\pi\right) \qquad z_{\psi} = -\omega_{\chi}\sin[\chi - (n-1)\pi]$$

- Constraint for the masses:

$$\begin{aligned} w_{\chi}^{2} &= w_{\chi}^{2}\left[\sin\left(n\,\chi\right)^{2} + \cos\left(n\,\chi\right)^{2}\right] \\ &= w_{\chi}^{2}\left[\psi_{\chi}^{(n)\,2} + \phi_{\chi}^{(n)\,2}\right] \\ &= z_{\psi}^{2} + y_{\phi}^{2}. \end{aligned}$$

Results

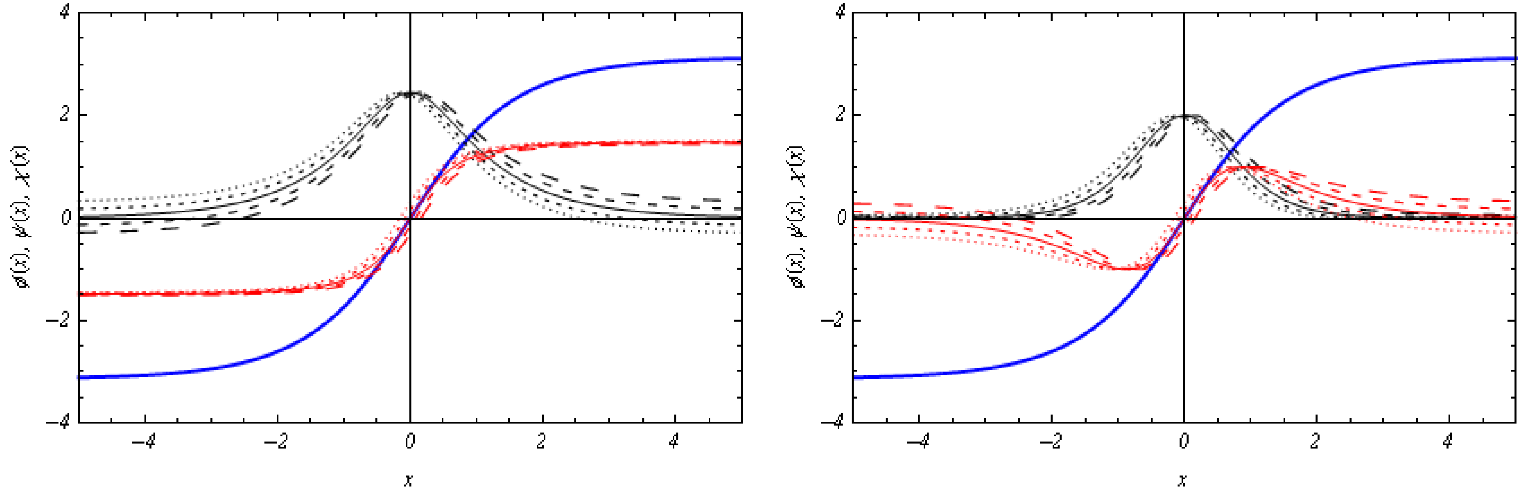
Potentials



These are the deformed potentials from the Sine Gordon model for the hyperbolic (first column) and trigonometric (second column) deformations.

Results

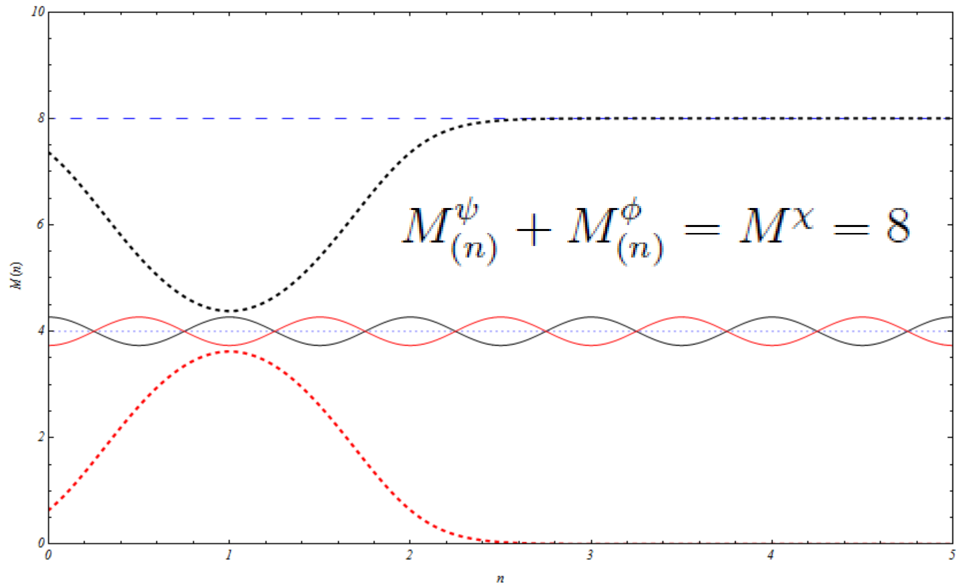
Defects



Graphics showing defects from hyperbolic (left side) and trigonometric (right side) deformations. The Sine Gordon's model initial defect (chi) is plotted as the blue line, whereas the deformed defects can be seen on the black (psi) and red (phi) lines, respectively.

Deformation mechanism

Topological Mass



The dashed blue line corresponding to the value 8, is the mass of the initial topological defect chi. The black and red solid lines correspond to masses of defects psi and phi, respectively, for the case of trigonometric deformation. Black and red Dotted lines correspond to the masses of defects psi and phi, respectively, for the hyperbolic case. We see that in both cases the sum of the masses of defects results in the mass of the initial deformed defect.

Final Comments

- Possibility of recovering the initial defect.

- Set constrain relations involving topological masses of the corresponding deformed defects in cyclic chains.

- Now that we have w_χ , y_ϕ e z_ψ we can get new solutions for different models in scenarios like the usual (1 + 3) and in brane world.

Thank you